Lecture 9

Fitting and Matching

- Problem Formulation
- Least Squares Methods
- RANSAC
- Hough Transforms
- Multi-model fitting
- Fitting helps matching

Reading:

[HZ] Chapter 4 “Estimation - 2D projective transformation”

[HZ] Chapter 11 “Computation of the fundamental matrix F”

[FP] Chapter 10 “Grouping and Model Fitting”

Fitting

Goals:

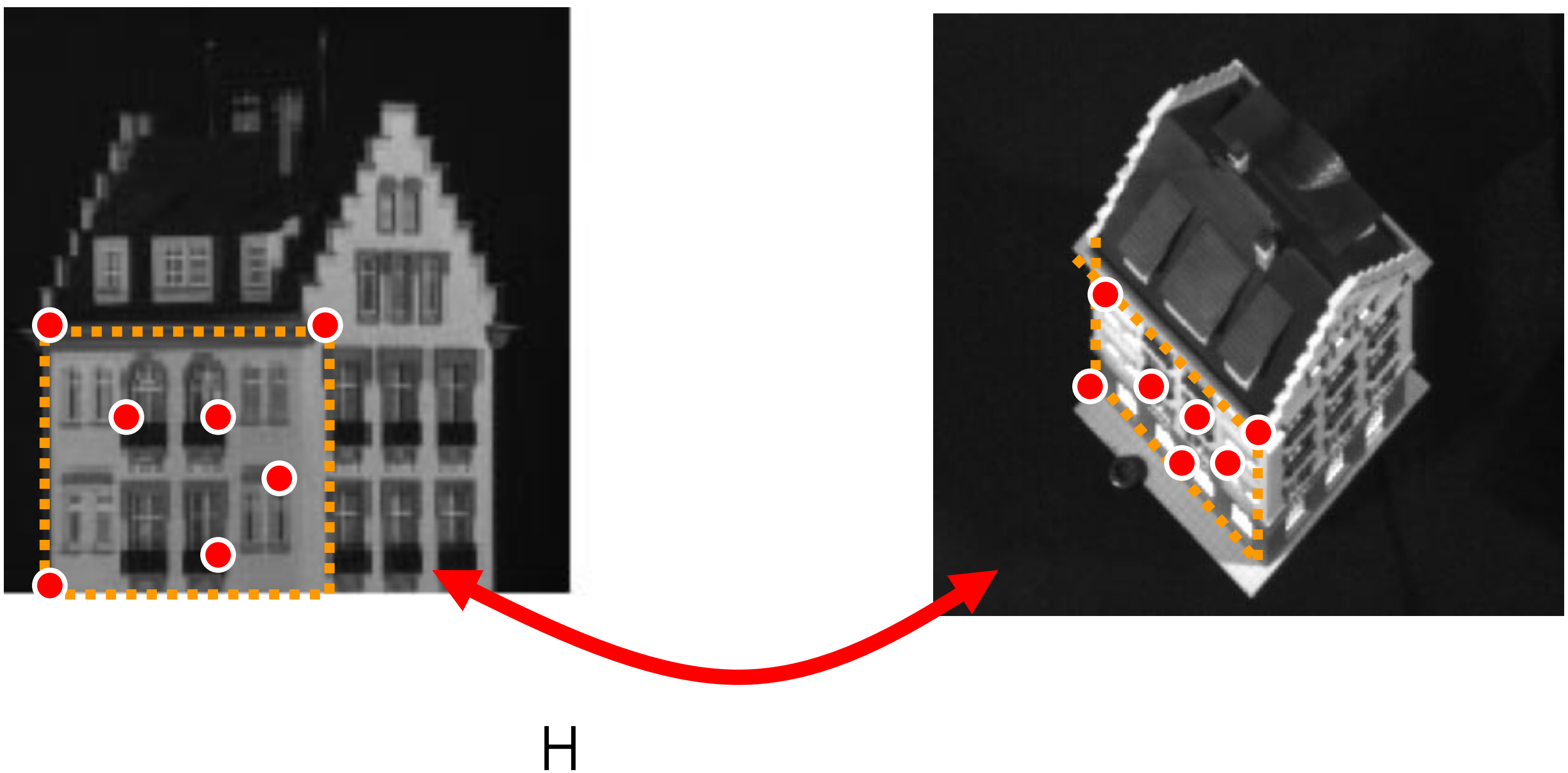
- Choose a parametric model to fit a certain quantity from data
- Estimate model parameters

- Lines
- Curves
- Homographic transformations
- Fundamental matrices
- Shape models

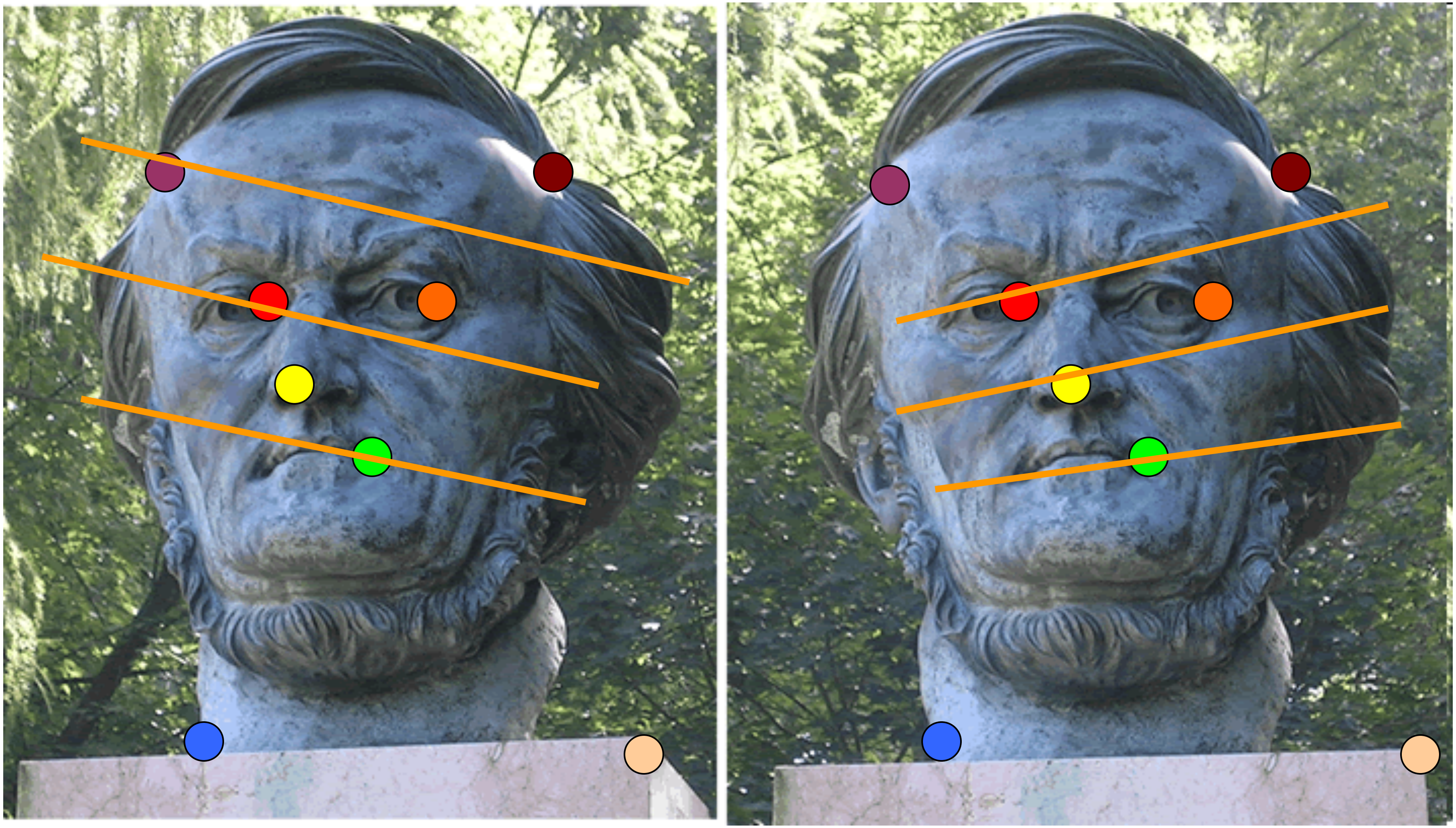
Example: fitting lines

(for computing vanishing points)

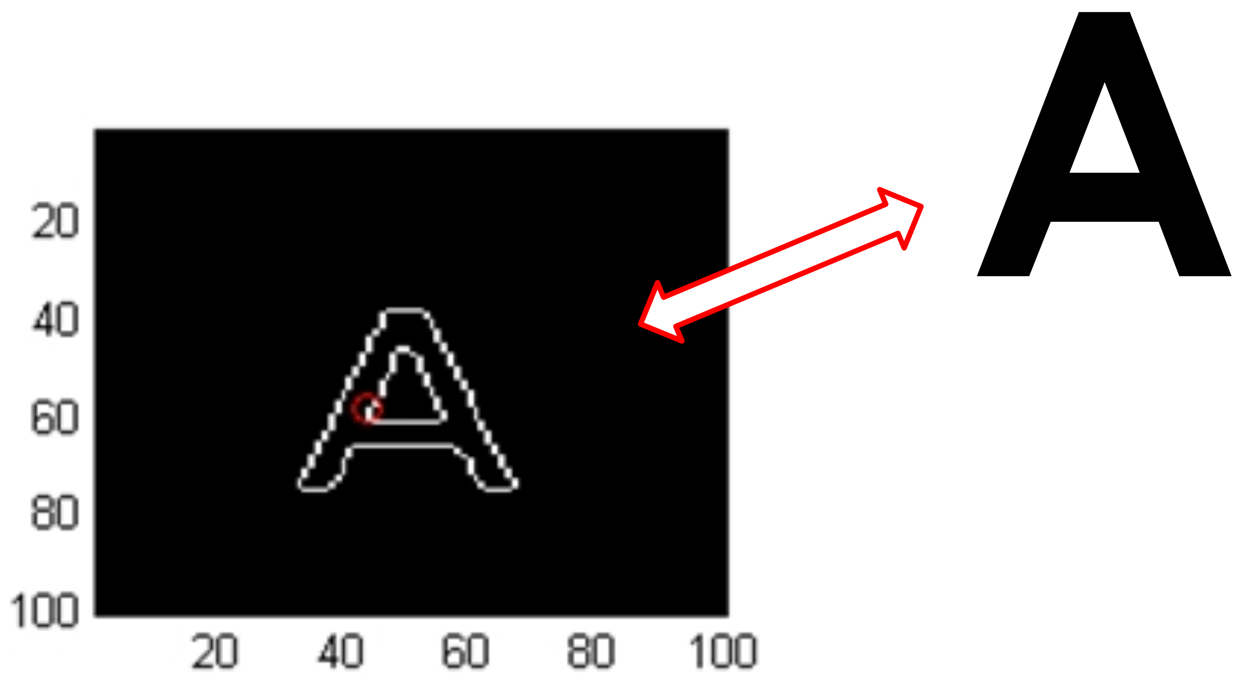
Example: Estimating an homographic transformation



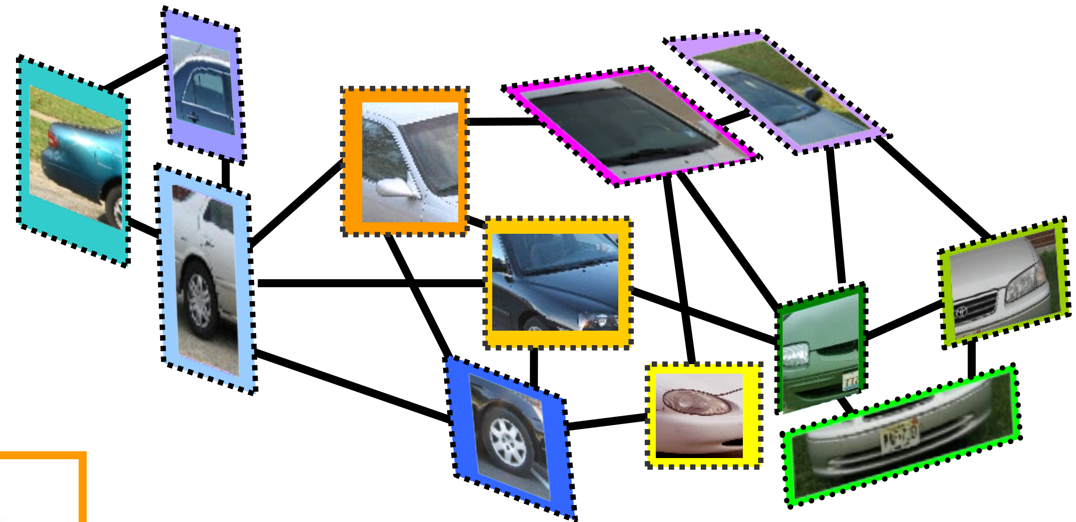
Example: Estimating F



Example: fitting a 2D shape template



Example: fitting a 3D object model



Fitting, matching and recognition are interconnected problems

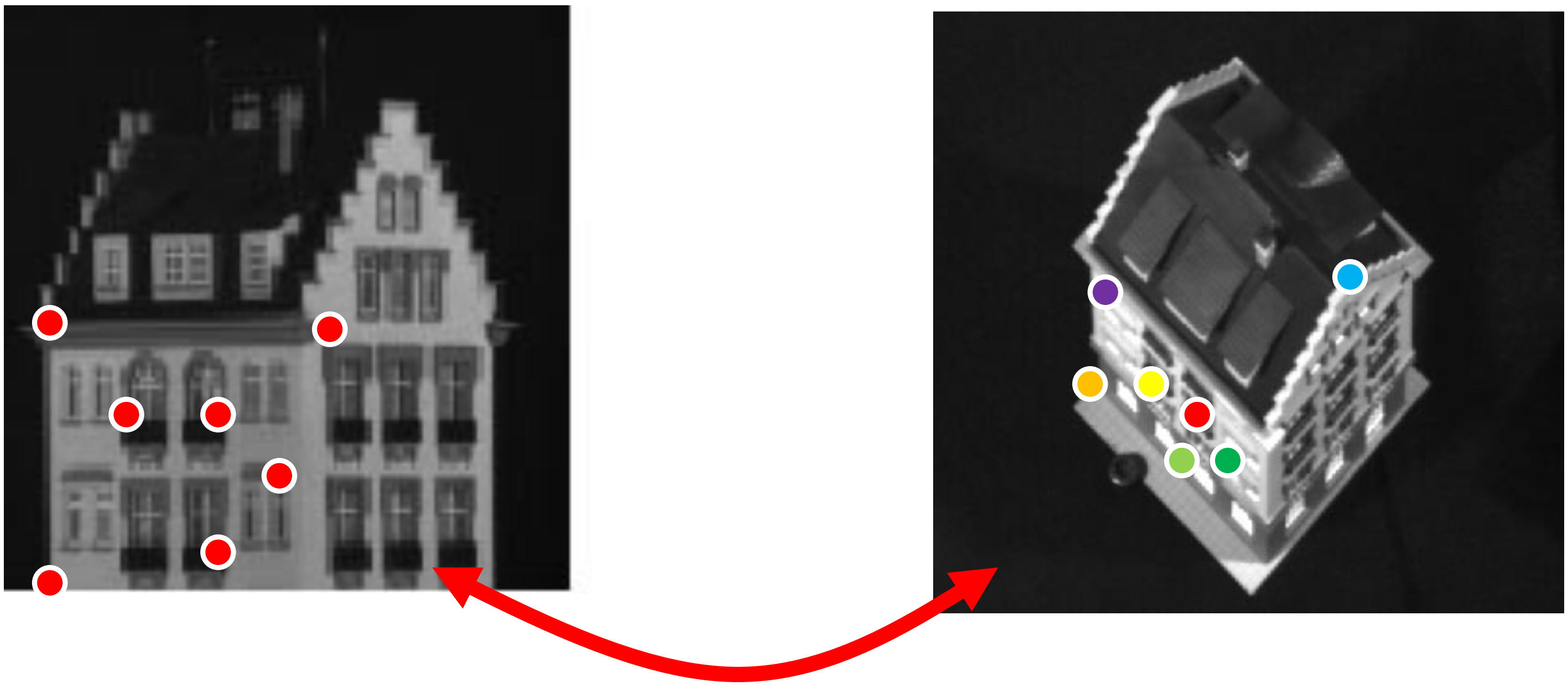
Fitting

Critical issues:

- noisy data
- outliers
- missing data
- Intra-class variation

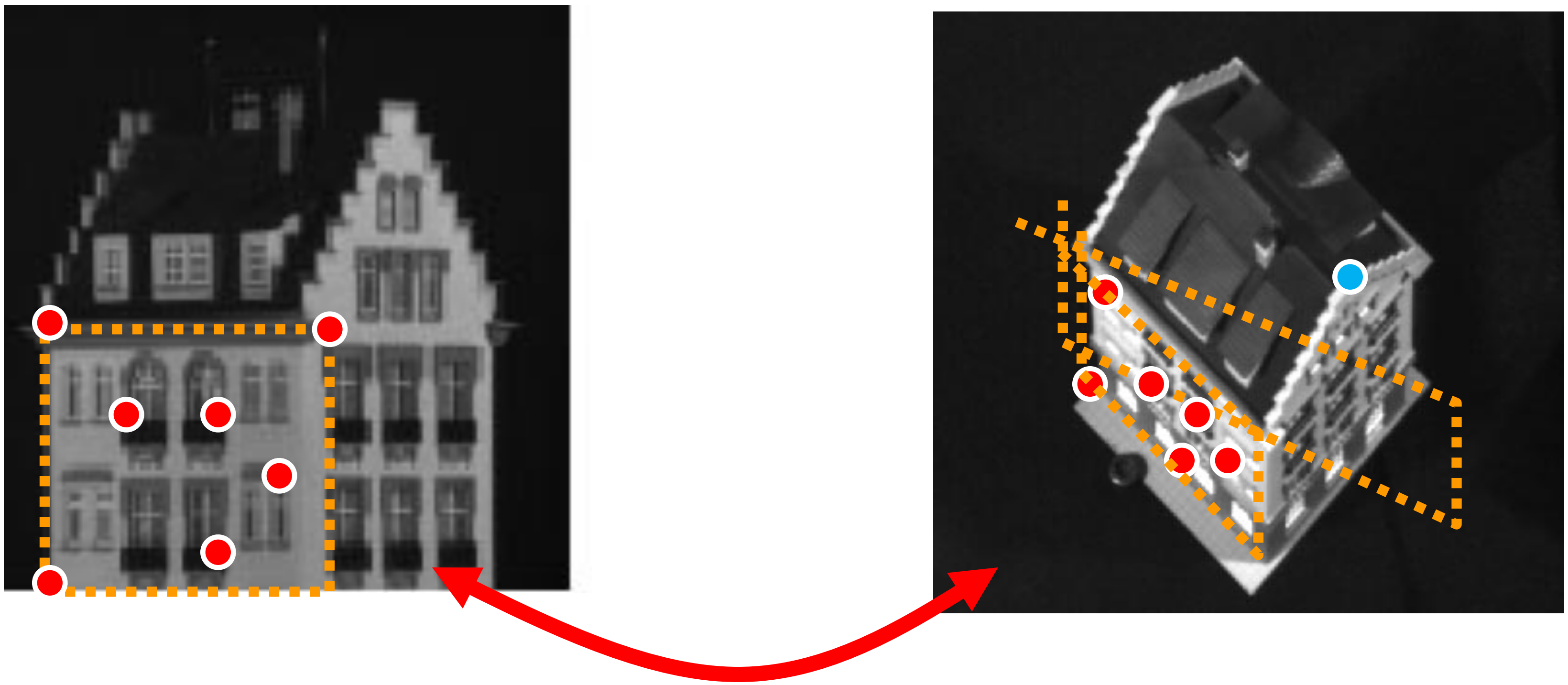
Critical issues: noisy data

Critical issues: outliers



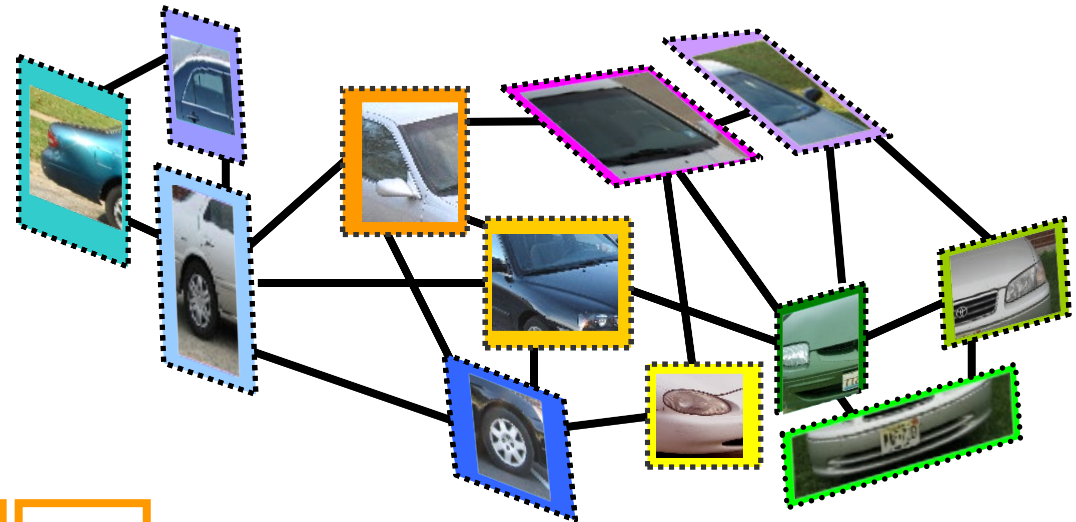
What's H?

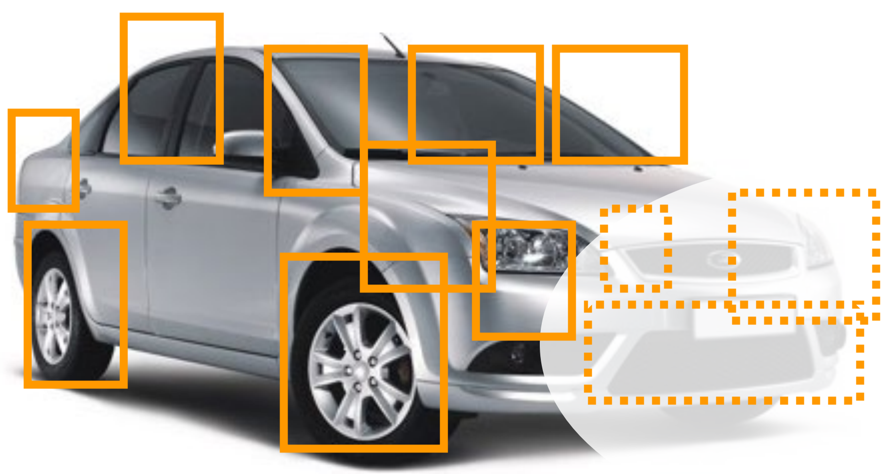
Critical issues: outliers



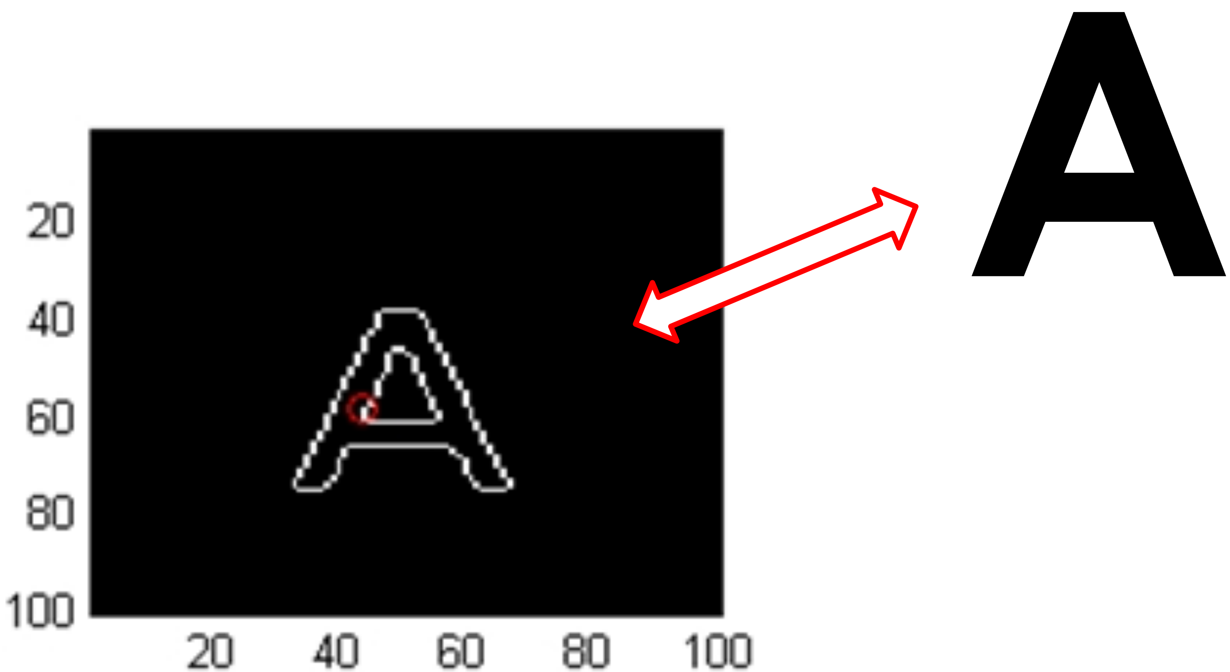
What's H? ☹

Critical issues: missing data (occlusions)





Critical issues: noisy data (intra-class variability)



Fitting

Goal: Choose a parametric model to fit a certain quantity from data

Techniques:

- Least square methods
- RANSAC
- Hough transform
- EM (Expectation Maximization) [not covered]

Least squares methods
- fitting a line -

- Data: $(x_1, y_1), \ldots, (x_n, y_n)$

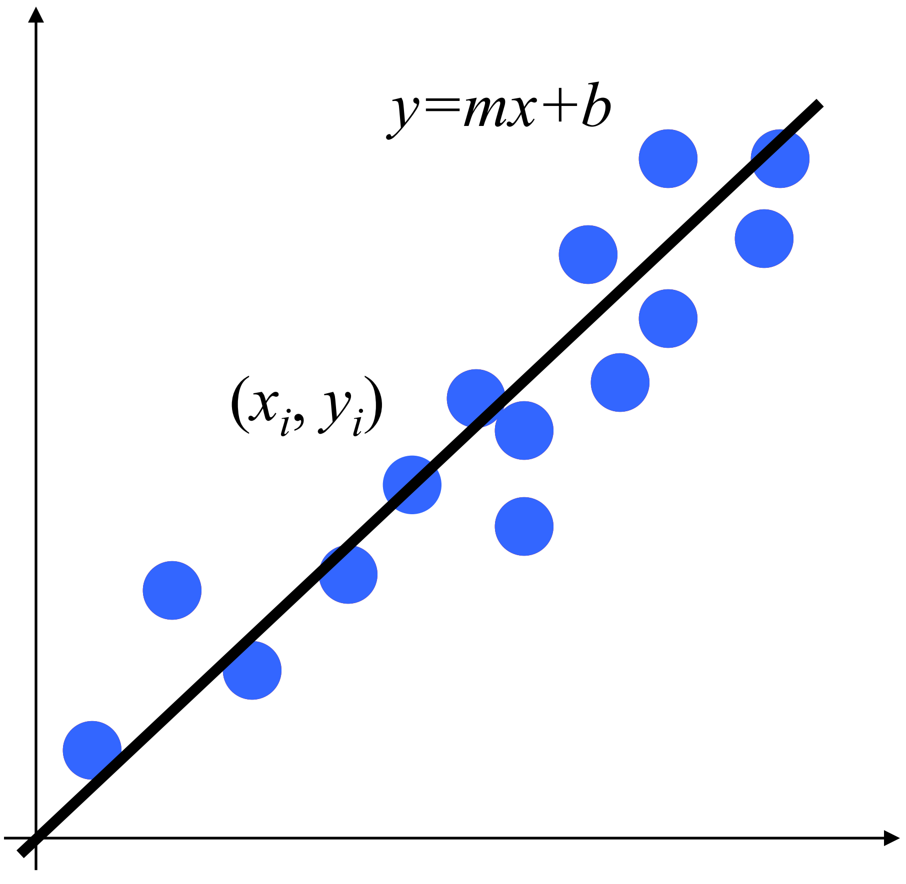
- Model of the line:

  $$y_i - m x_i - b = 0 \qquad \text{[Eq. 1]}$$

- Parameters: *m, b*

- Find (m, b) to minimize fitting error (**residual**):

  $$\boxed{E = \sum_{i=1}^{n} (y_i - m x_i - b)^2} \qquad \text{[Eq. 2]}$$



Least squares methods
- fitting a line -

$$\boxed{E = \sum_{i=1}^{n} (y_i - m x_i - b)^2}$$ [Eq. 2]

$$E = \sum_{i=1}^{n} \left(y_i - \begin{bmatrix} x_i & 1 \end{bmatrix} \begin{bmatrix} m \\ b \end{bmatrix} \right)^2 = \left\| \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix} - \begin{bmatrix} x_1 & 1 \\ \vdots & \vdots \\ x_n & 1 \end{bmatrix} \begin{bmatrix} m \\ b \end{bmatrix} \right\|^2 = \left\| Y - Xh \right\|^2$$ [Eq. 3]

$$= (Y - Xh)^T (Y - Xh) = Y^T Y - 2(Xh)^T Y + (Xh)^T (Xh)$$ [Eq. 4]

Find $h = [m, b]^T$ that minimizes E

$$\frac{dE}{dh} = -2X^T Y + 2X^T Xh = 0$$ [Eq. 5]

$$X^T Xh = X^T Y$$ [Eq. 7]

Normal equation

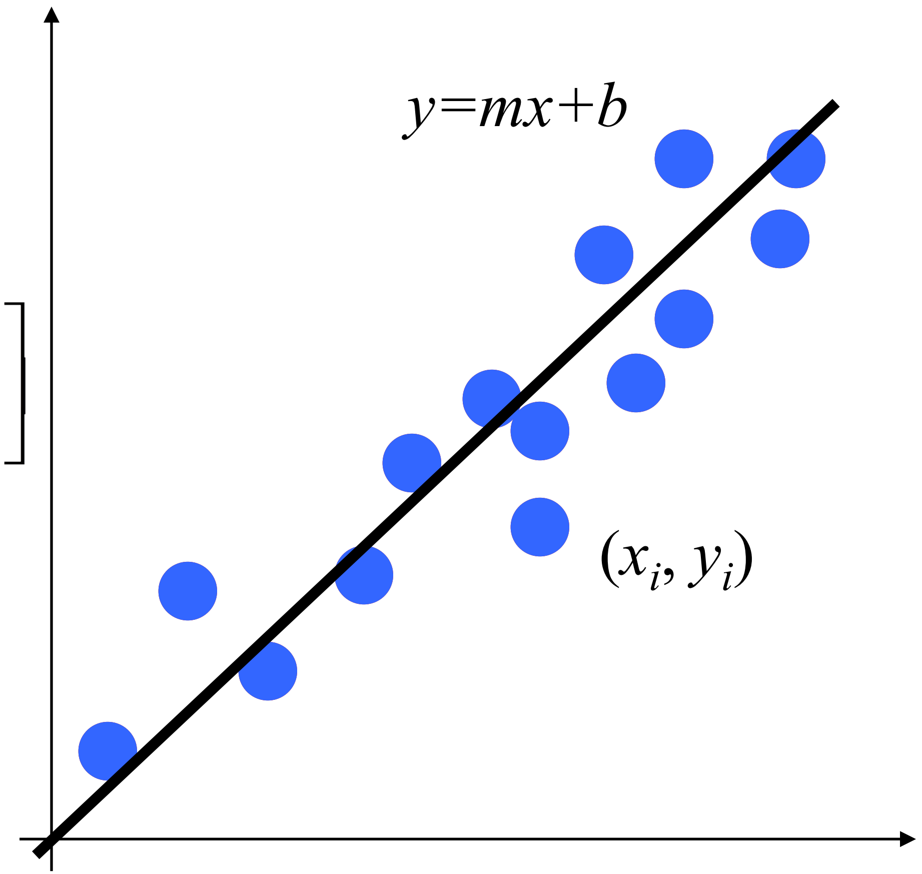
$$\boxed{h = \left(X^T X \right)^{-1} X^T Y}$$ [Eq. 6]

Least squares methods
- fitting a line -

$$E = \sum_{i=1}^{n} (y_i - mx_i - b)^2$$

$$\boxed{h = \left(X^T X\right)^{-1} X^T Y} \qquad h = \begin{bmatrix} m \\ b \end{bmatrix}$$

[Eq. 6]



Issues?

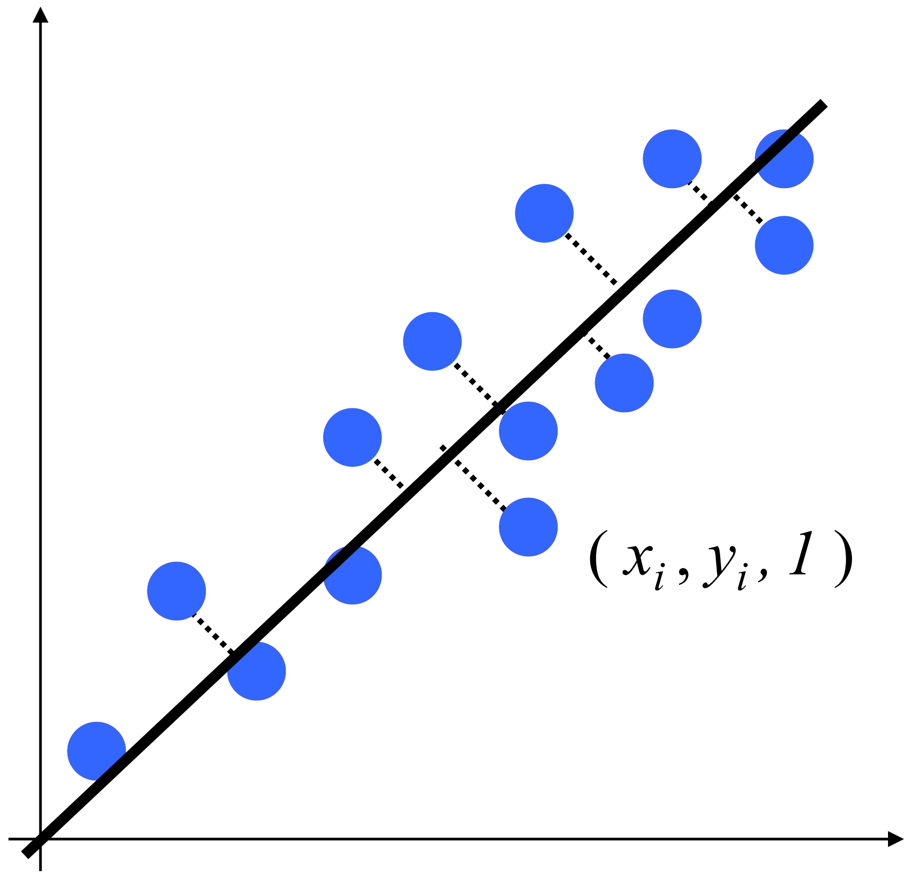
- Fails completely for vertical lines

Least squares methods
- fitting a line -

- Distance between point $(x_i, y_i, 1)$ and line (a, b, d)

  ax + by = d

- Find (a, b, d) to minimize the sum of squared perpendicular distances



[Eq. 8]

$$\boxed{E = \sum_{i=1}^{n} (ax_i + by_i + d)^2}$$

$$\underbrace{A}_{\text{data}}\ \underbrace{h}_{\text{model parameters}} = 0$$

[Eq. 9]

Least squares methods
- fitting a line -

$$\text{A h} = 0 \qquad \text{A is rank deficient}$$

$$\text{Minimize} \, \|\text{A h}\| \quad \text{subject to} \quad \|\text{h}\| = 1$$

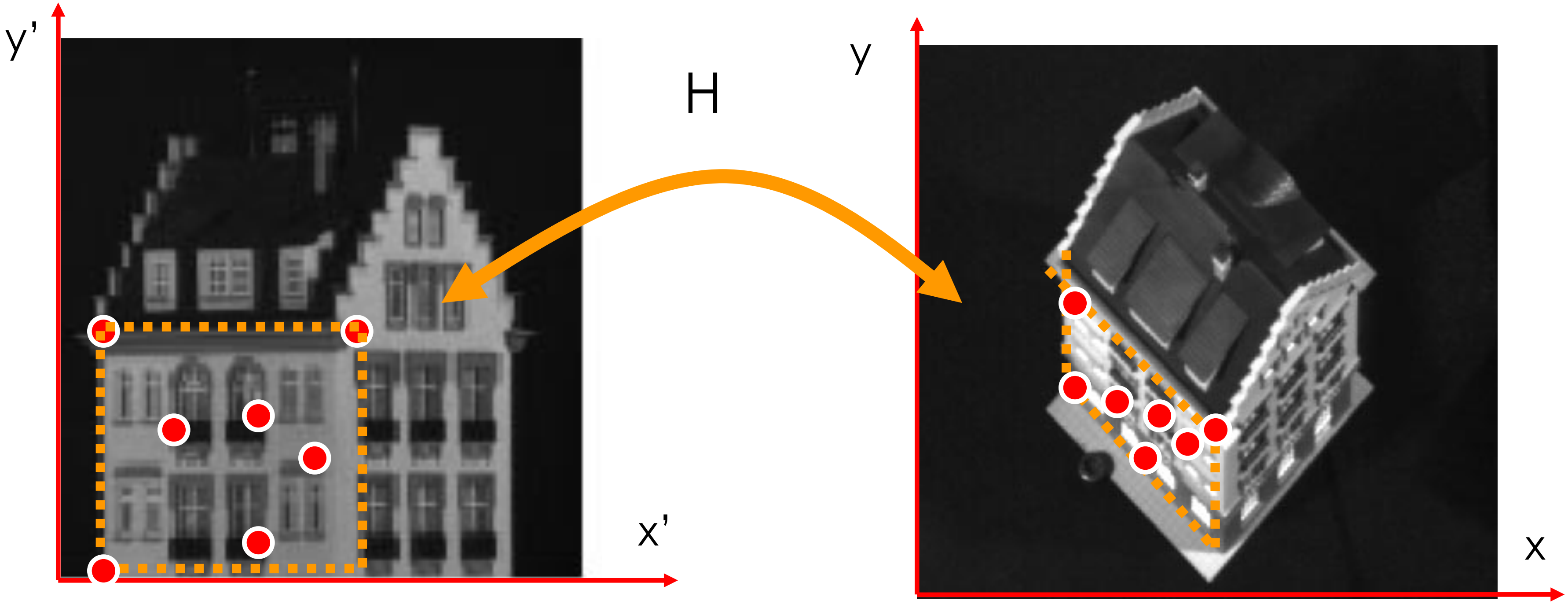
$$A = UDV^T$$

$$\text{h} = \text{last column of V}$$

See [HZ], sec. A5.3 - page 593

Least squares methods

- fitting an homography -

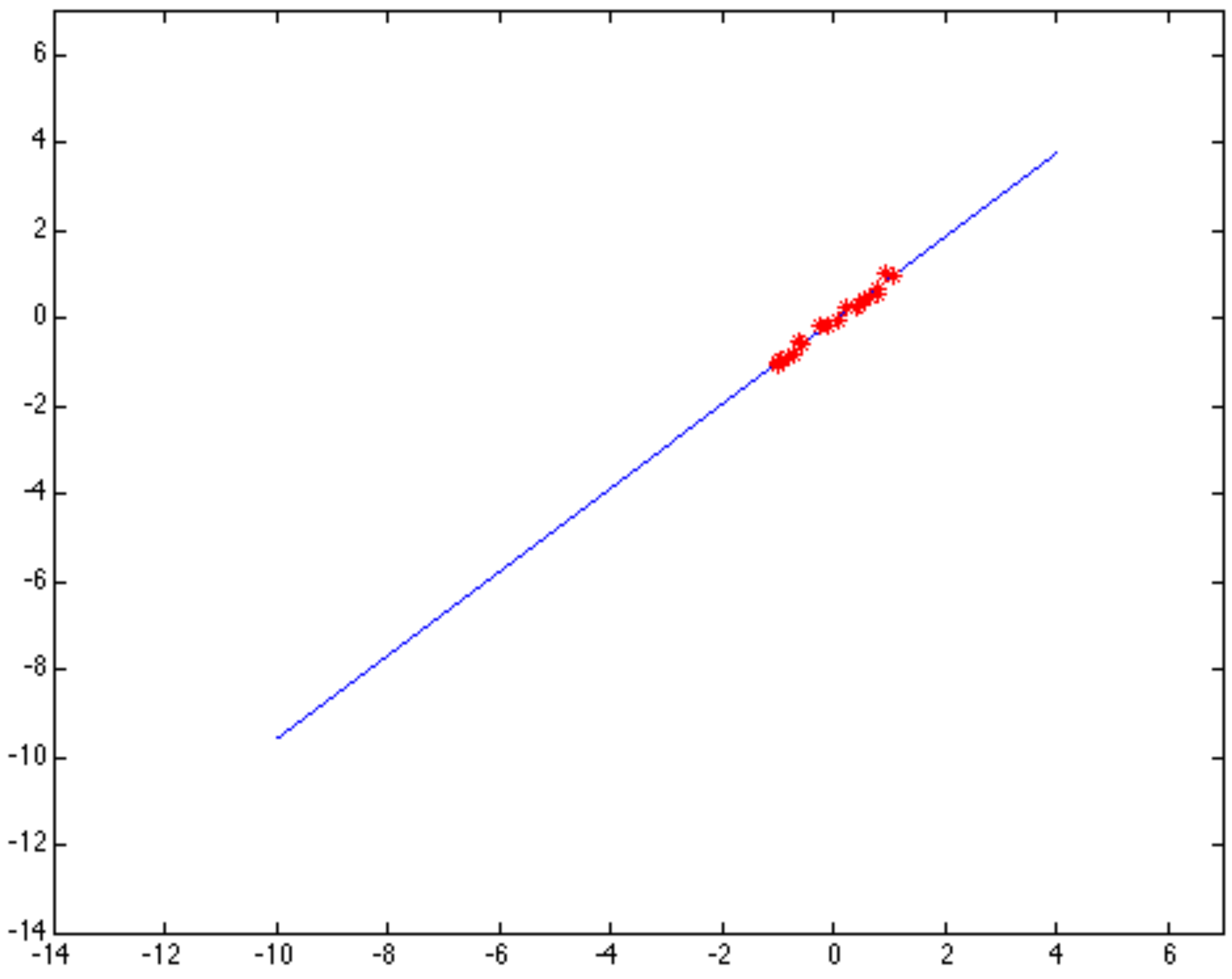


$$\boxed{A}\,\boxed{h} = 0 \qquad \text{[Eq. 10]}$$

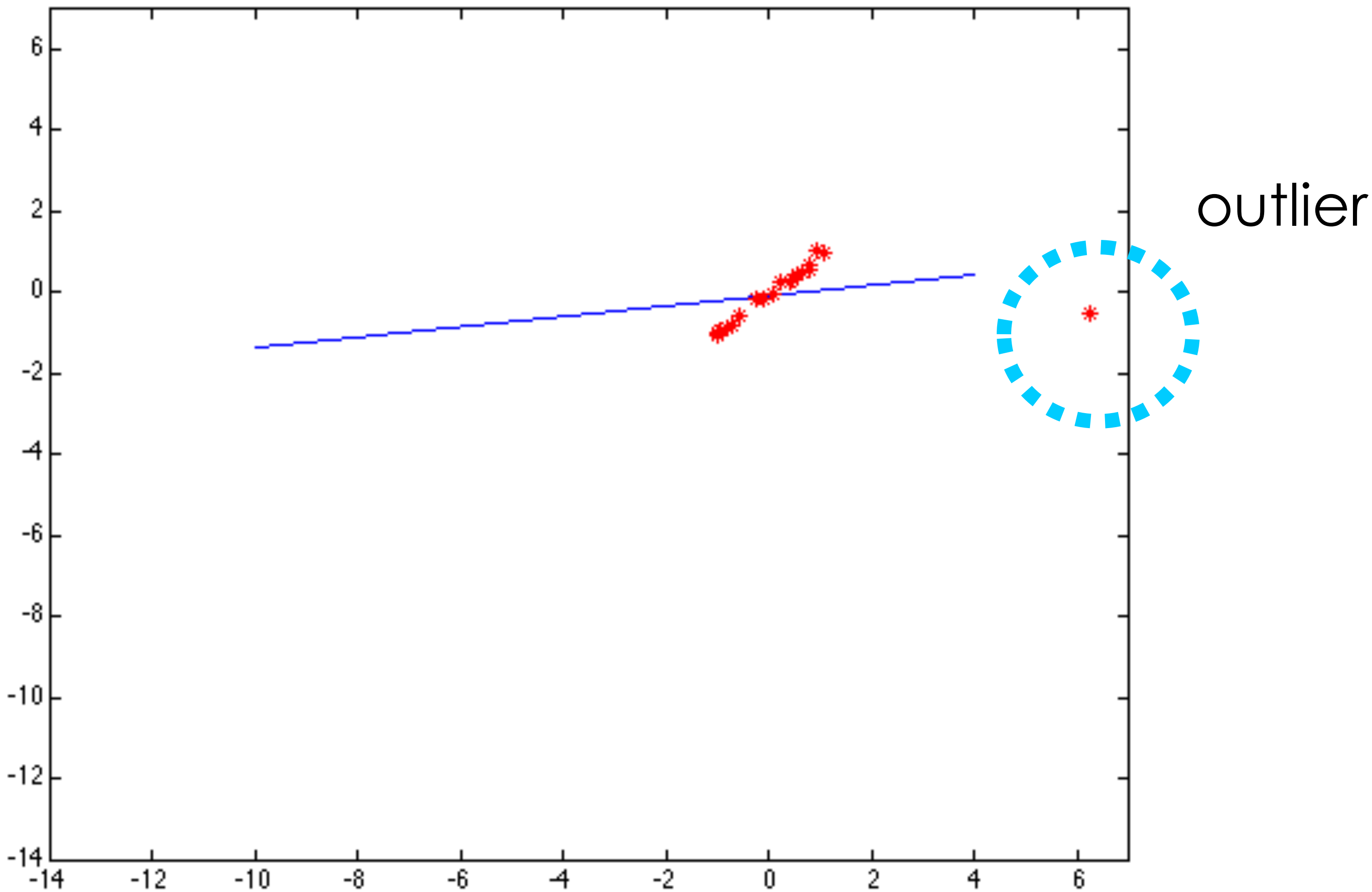
data   model parameters

See HZ
- Sec 4.1 for details (DLT algorithm)
- Sec 4.1.2 (or APPENDIX)

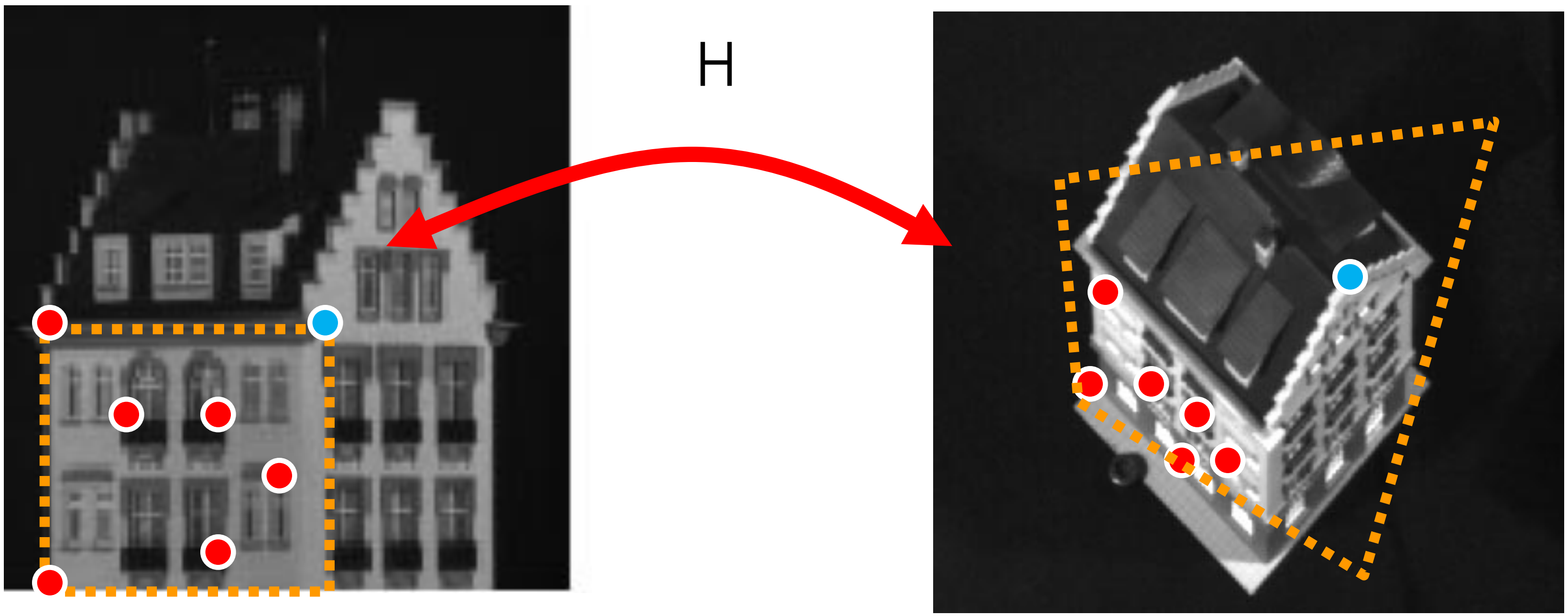
Least squares: Robustness to noise



Least squares: Robustness to noise



Critical issues: outliers



CONCLUSION: Least square is not robust w.r.t. outliers

Fitting

Goal: Choose a parametric model to fit a certain quantity from data

Techniques:

- Least square methods
- RANSAC
- Hough transform

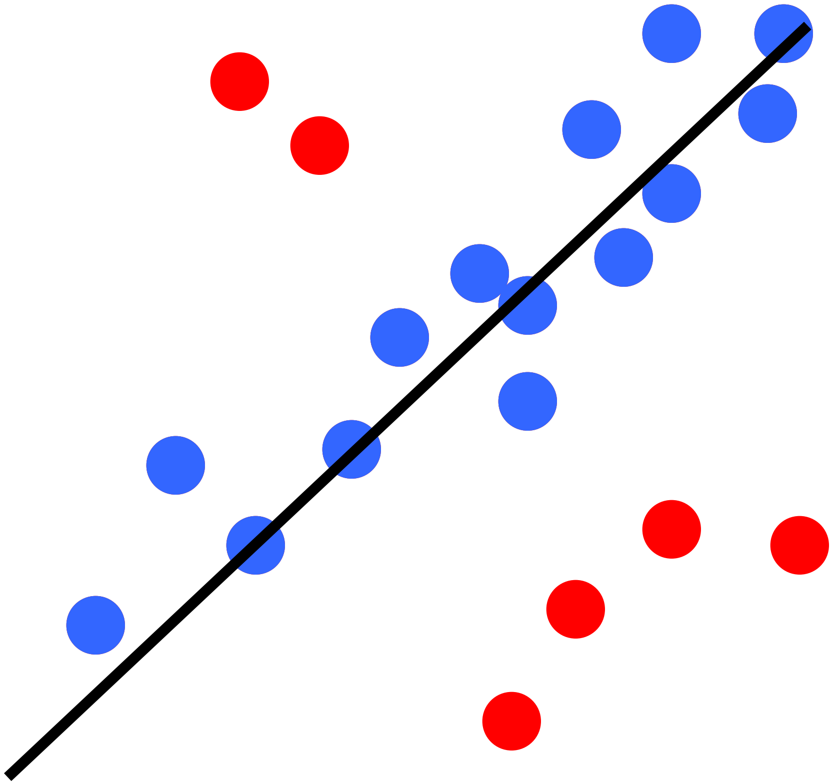
Basic philosophy
(voting scheme)

- Data elements are used to vote for one (or multiple) models

- Robust to outliers and missing data

- Assumption 1: Noisy data points will not vote consistently for any single model ("few" outliers)
- Assumption 2: There are enough data points to agree on a good model ("few" missing data)

Example: Line fitting

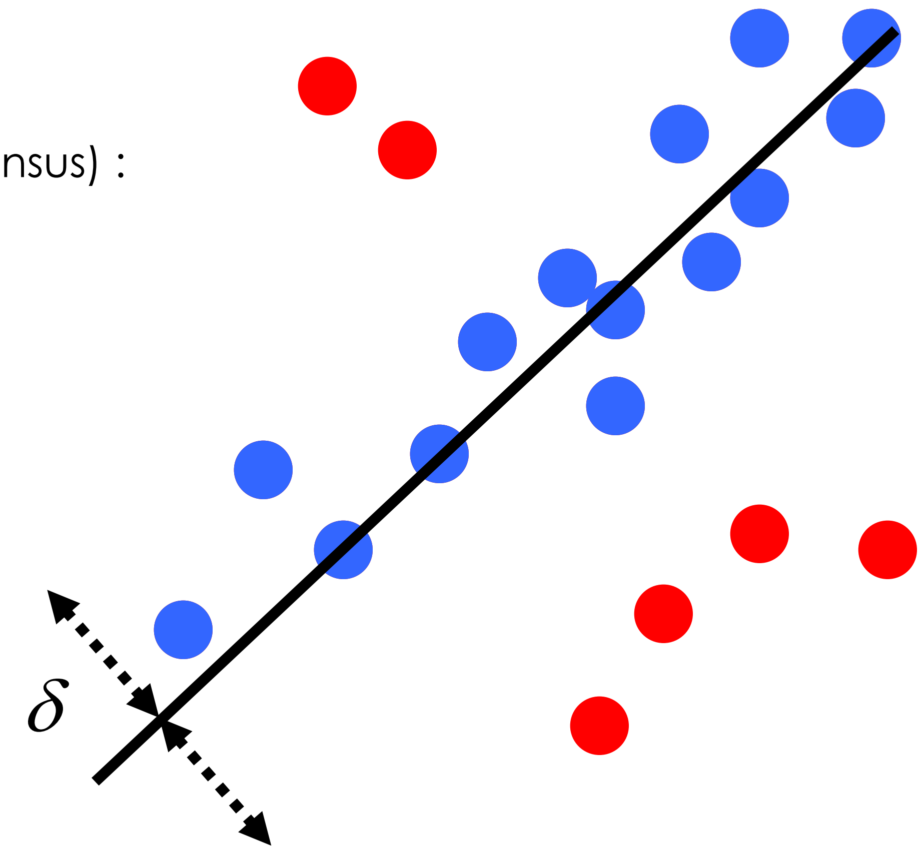


- Enough “good” data points supporting the line model in presence of noise
- “Few” outliers compared to the “good” data points – these few outliers won’t “consistently” vote for a line model

RANSAC

(RANdom SAmple Consensus) :

Fischler & Bolles in '81.



$$\pi : \mathbf{P} \rightarrow \{\mathbf{I}, \mathbf{O}\} \qquad \min_{\pi} |O|$$

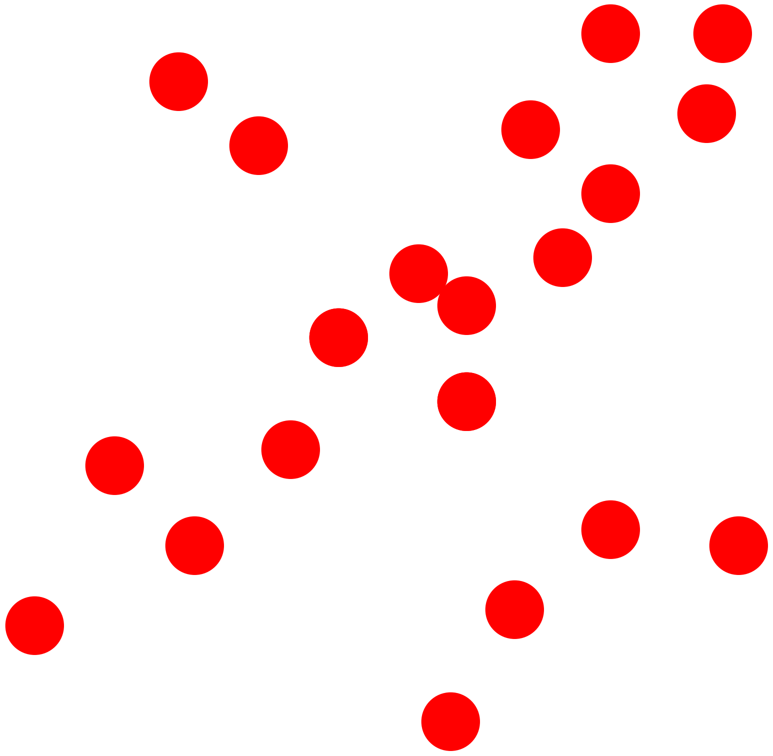
Model parameters a,b,d

such that:

$$r(I, h) < \delta, \quad \forall I \in \mathbf{I} \qquad r(I, h) = residual \boxed{= \sum_{i=1}^{n} (ax_i + by_i + d)^2}$$

[Eq. 12]

RANSAC



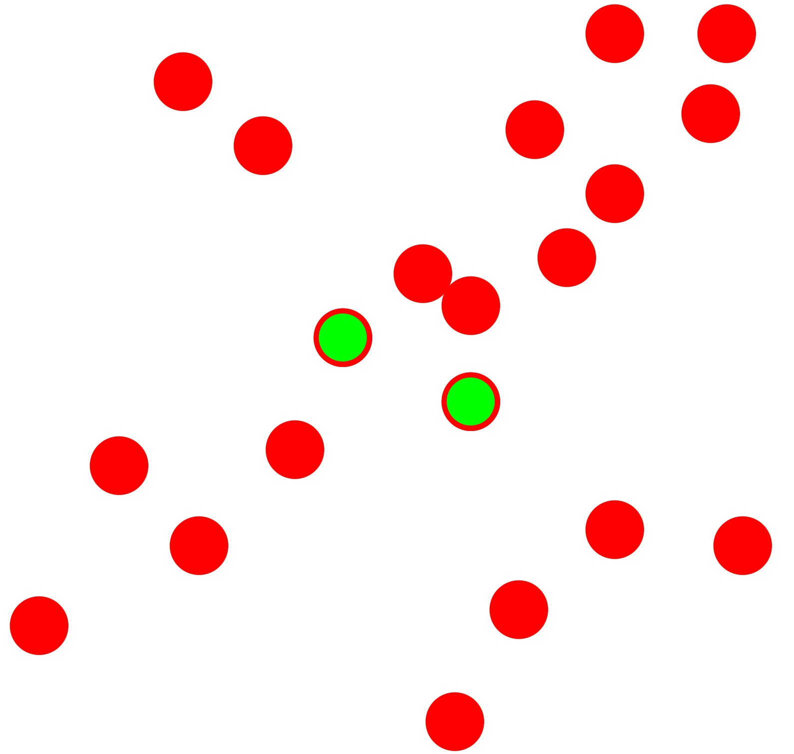
P = Sample set = set of points in 2D

Algorithm:

1. Select random sample of minimum required size to fit model
2. Compute a putative model from sample set
3. Compute the set of inliers to this model from whole data set

Repeat 1-3 until model with the most inliers over all samples is found

RANSAC



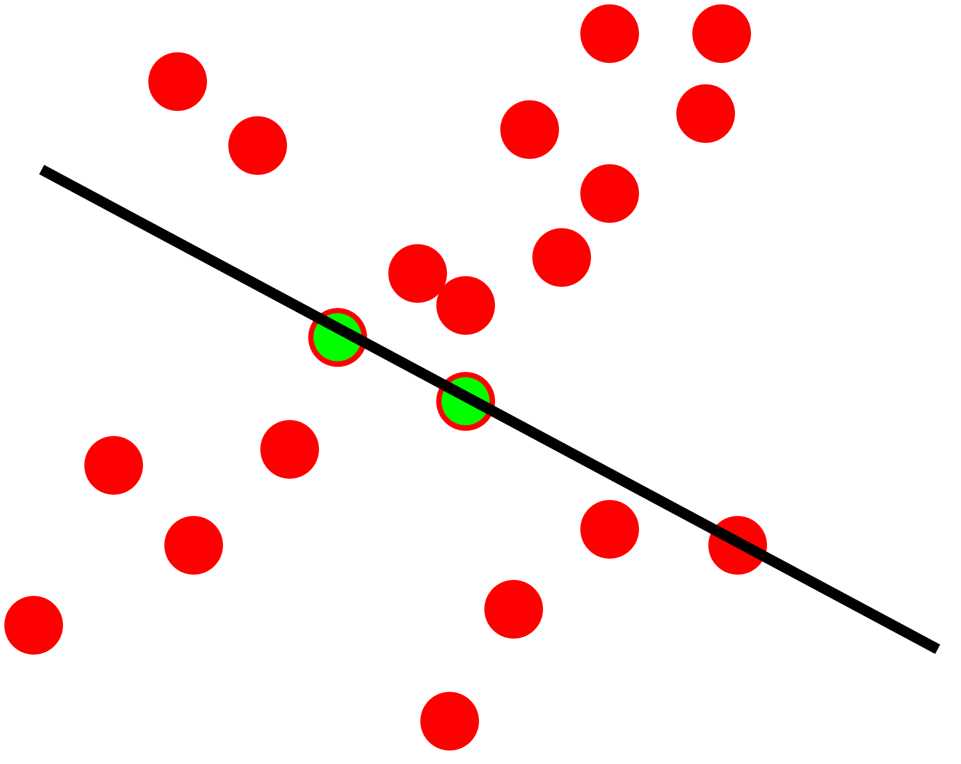
P = Sample set = set of points in 2D

Algorithm:

1. Select random sample of minimum required size to fit model [?]
2. Compute a putative model from sample set
3. Compute the set of inliers to this model from whole data set

Repeat 1-3 until model with the most inliers over all samples is found

RANSAC



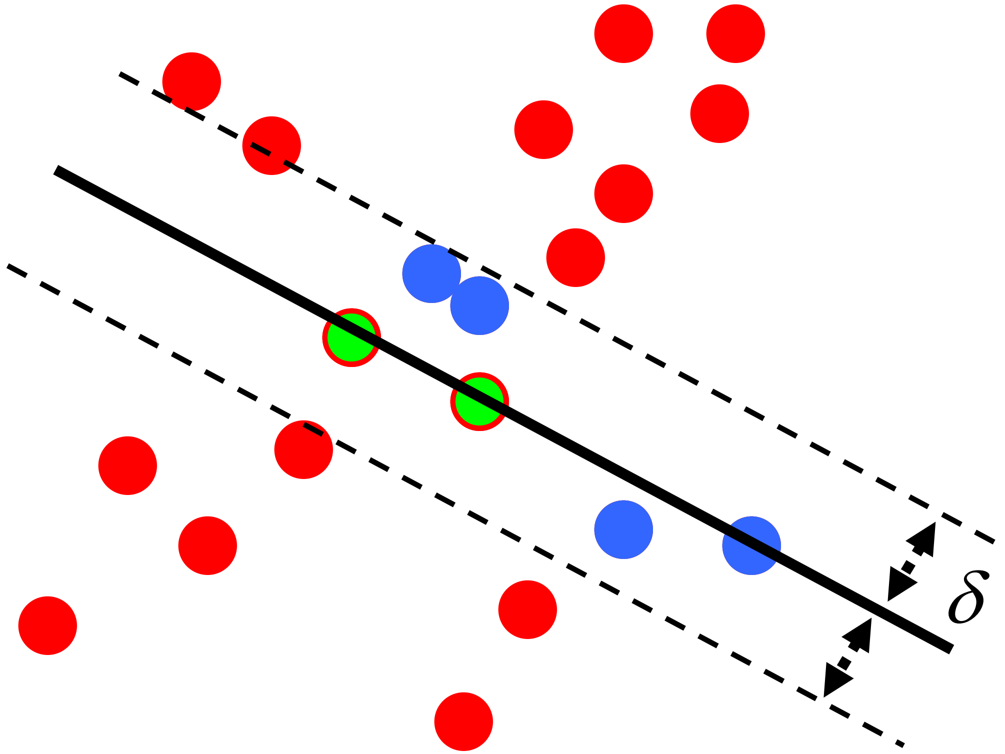
P = Sample set = set of points in 2D

Algorithm:

1. Select random sample of minimum required size to fit model [?]
2. Compute a putative model from sample set
3. Compute the set of inliers to this model from whole data set

Repeat 1-3 until model with the most inliers over all samples is found

RANSAC



P = Sample set = set of points in 2D

$$|\mathbf{O}| = ? = 14$$

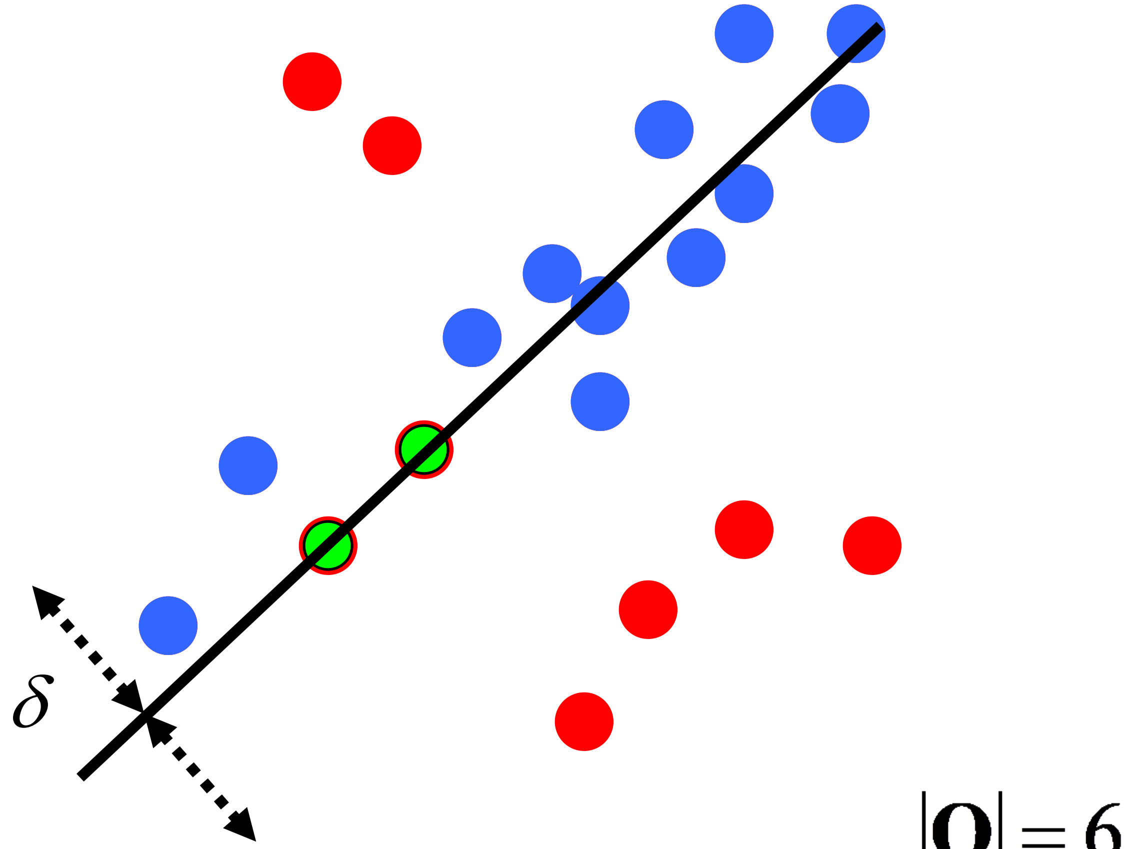
$$|\mathbf{I}| = ? = 6$$

Algorithm:

1. Select random sample of minimum required size to fit model [?]
2. Compute a putative model from sample set
3. Compute the set of inliers to this model from whole data set

Repeat 1-3 until model with the most inliers over all samples is found

RANSAC



$$|\mathbf{O}| = 6$$

$$|\mathbf{I}| = 14$$

Algorithm:

1. Select random sample of minimum required size to fit model [?]
2. Compute a putative model from sample set
3. Compute the set of inliers to this model from whole data set

Repeat 1-3 until model with the most inliers over all samples is found

How many samples?

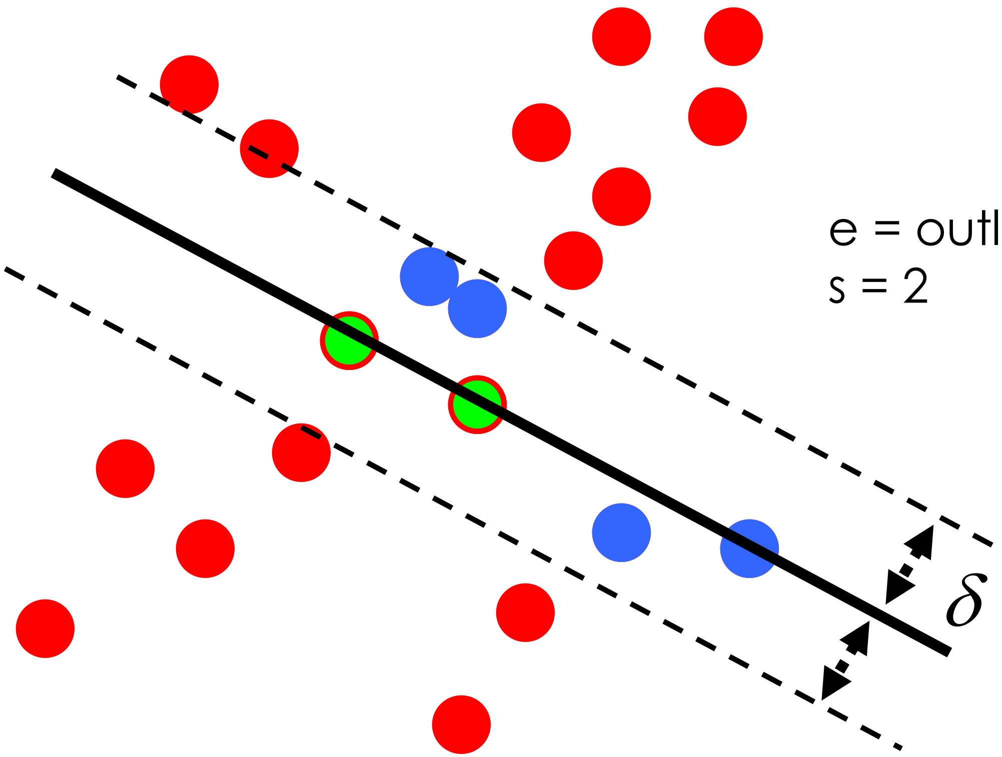
- Computationally unnecessary (and infeasible) to explore the entire sample space

- **N samples are sufficient**
- N = number of samples required to ensure, with a probability p, that at least one random sample produces an inlier set that is free from "real" outliers

- Function of *s* and *e*:
  - *e* = outlier ratio
  - s = minimum number of data points needed to fit the model

- Usually, p=0.99

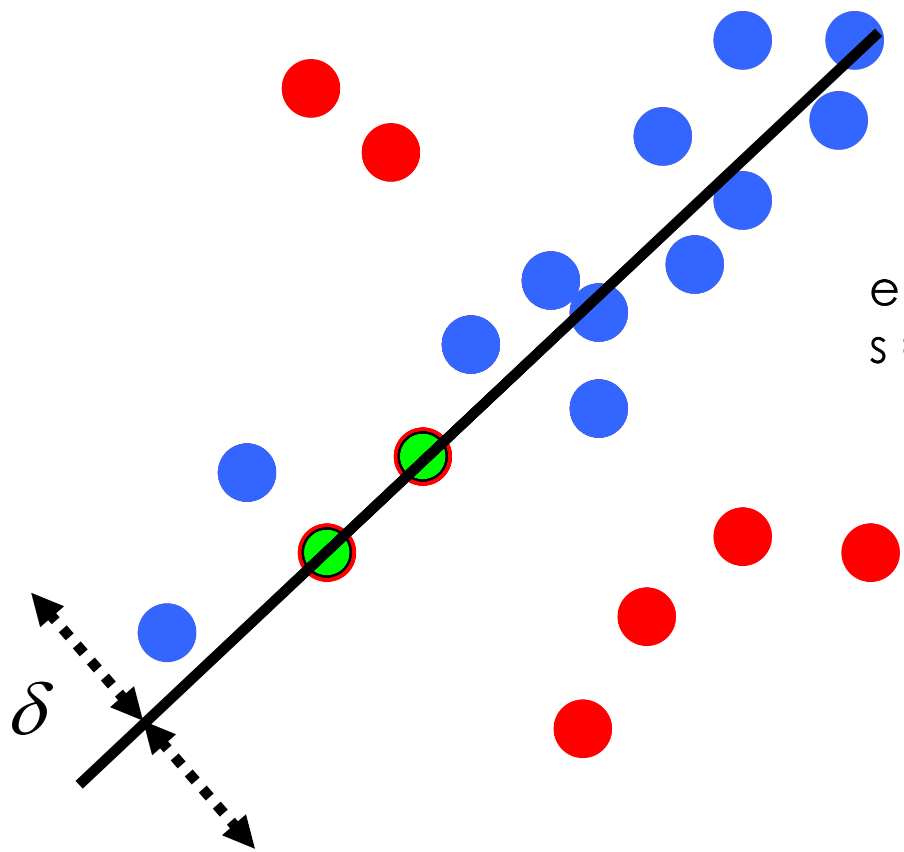
Example



e = outlier ratio is 6/20 = 30%
s = 2

- Here a random sample is given by two green points
- The estimated inlier set is given by the green+blue points
- How many "real" outliers we have here? 2

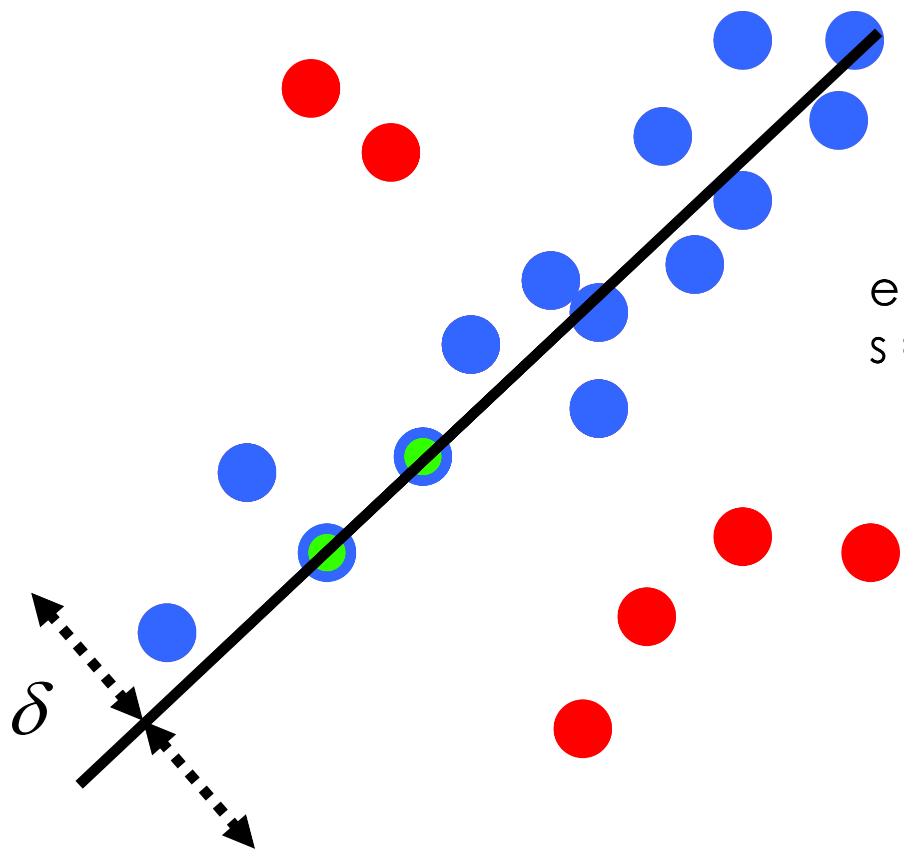
Example



e = outlier ratio is 6/20 = 30%
s = 2

- Random sample is given by two green points
- The estimated inlier set is given by the green+blue points
- How many "real" outliers we have here? 0

Example



e = outlier ratio is 6/20 = 30%
s = 2

N is the number of times we need to sample my data (and thus repeat the steps 1-3 in the previous slides) before I find the configuration above with probability *p*. Again this is function of *e* and *s* as well.

How many samples?

- Number N of samples required to ensure, with a probability p, that at least one random sample produces an inlier set that is free from "real" outliers for a given s and e.
- E.g., p=0.99

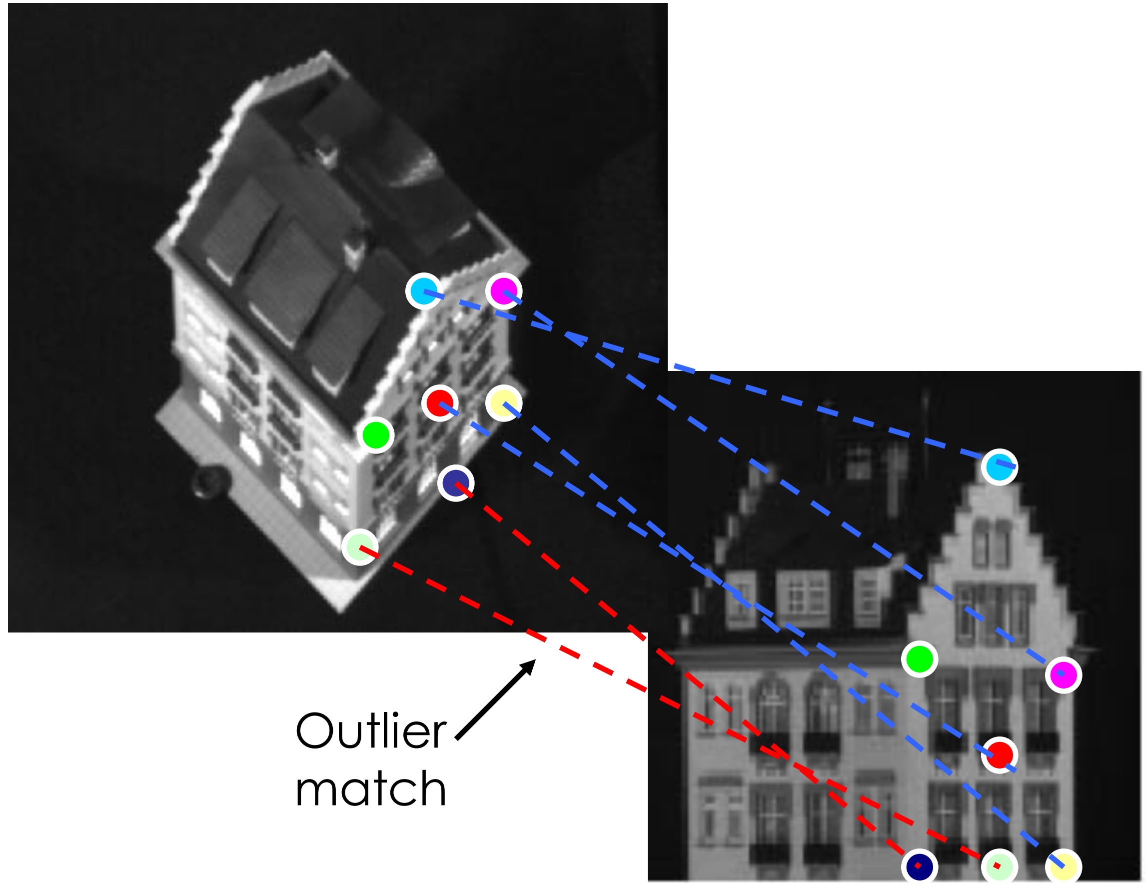
$$N = \log(1-p) / \log\left(1-(1-e)^s\right)$$ [Eq. 13]

	proportion of outliers e						
s	5%	10%	20%	25%	30%	40%	50%
2	2	3	5	6	$\boxed{7}$	11	17
3	3	4	7	9	11	19	35
4	3	5	9	13	17	34	72
5	4	6	12	17	26	57	146
6	4	7	16	24	37	97	293
7	4	8	20	33	54	163	588
8	5	9	26	44	78	272	1177

e = outlier ratio
s = minimum number needed to fit the model

Note: this table assumes "negligible" measurement noise

Estimating H by RANSAC



- H $\rightarrow$ 8 DOF
- Need 4 correspondences

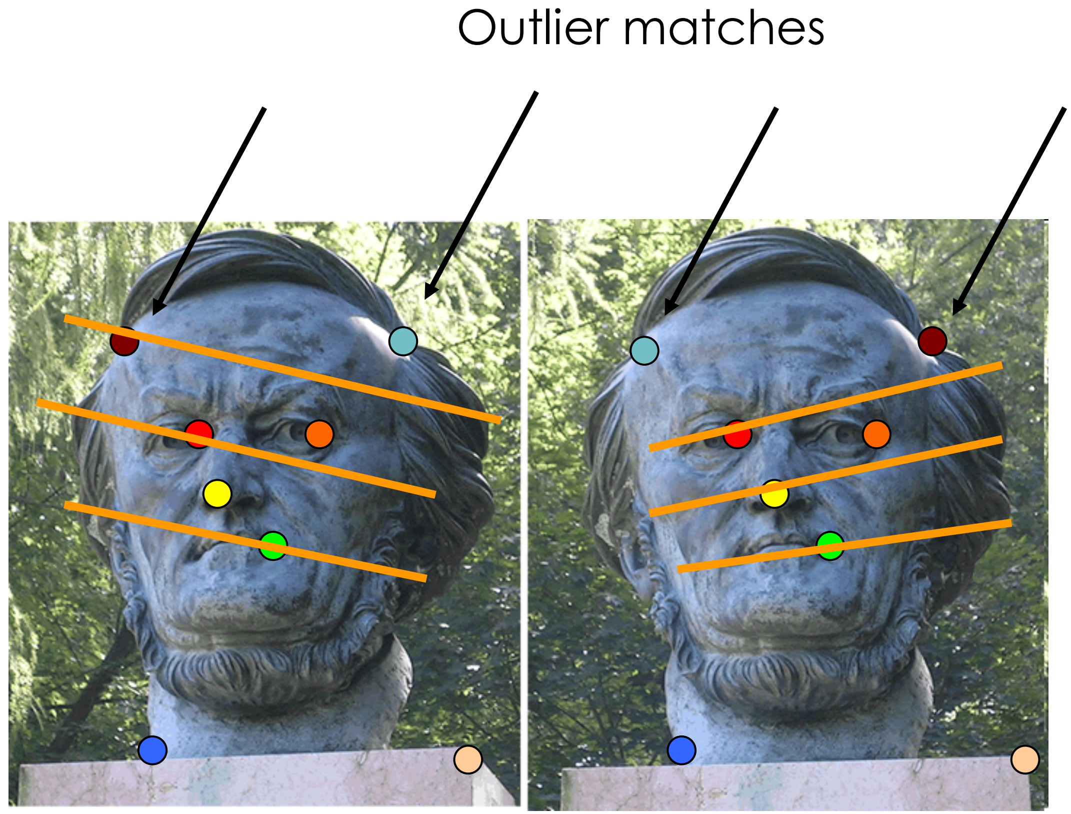
P = Sample set = set of matches between 2 images

Algorithm:

1. Select a random sample of minimum required size [?]
2. Compute a putative model from these
3. Compute the set of inliers to this model from whole sample space

Repeat 1-3 until model with the most inliers over all samples is found

Estimating F by RANSAC



- F → 7 DOF
- Need 8 correspondences

P = Sample set = set of matches between 2 images

Algorithm:

1. Select a random sample of minimum required size [?]
2. Compute a putative model from these
3. Compute the set of inliers to this model from whole sample space

Repeat 1-3 until model with the most inliers over all samples is found

RANSAC - conclusions

Good:

- Simple and easily implementable
- Successful in different contexts

Bad:

- Many parameters to tune
- Trade-off accuracy-vs-time
- Cannot be used if ratio inliers/outliers is too small

Fitting

Goal: Choose a parametric model to fit a certain quantity from data

Techniques:

- Least square methods
- RANSAC
- Hough transform

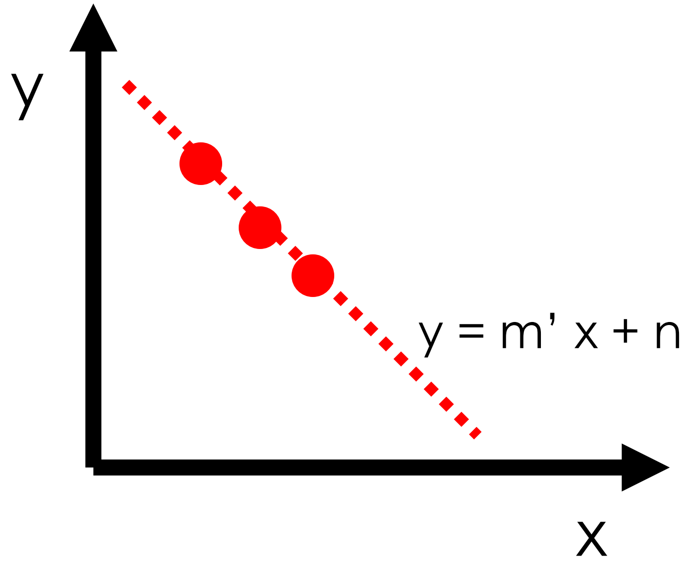
Hough transform

P.V.C. Hough, *Machine Analysis of Bubble Chamber Pictures,* Proc. Int. Conf. High Energy Accelerators and Instrumentation, 1959

Hough transform

P.V.C. Hough, *Machine Analysis of Bubble Chamber Pictures,* Proc. Int. Conf. High Energy Accelerators and Instrumentation, 1959

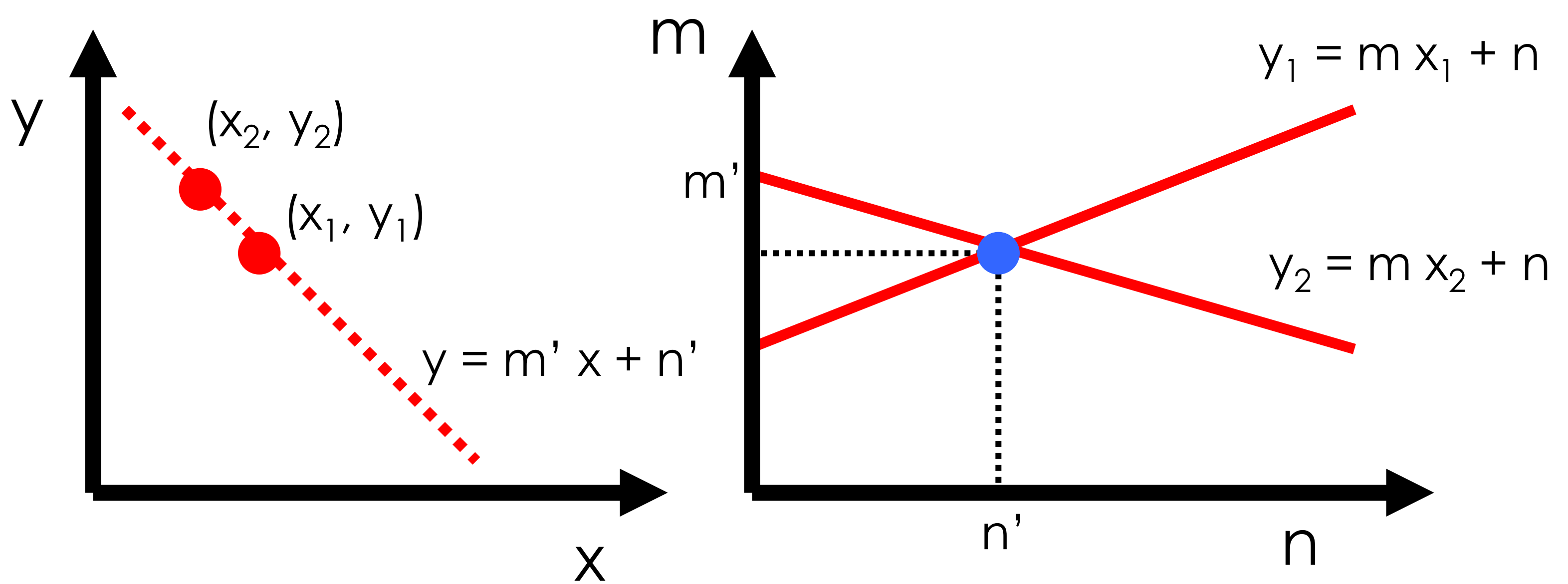
Given a set of points, find the line parameterized by *m,n* that explains the data points best: that is, *m* = *m'* and *n* = *n'*



Hough transform

P.V.C. Hough, *Machine Analysis of Bubble Chamber Pictures,* Proc. Int. Conf. High Energy Accelerators and Instrumentation, 1959

Given a set of points, find the line parameterized by m,n that explains the data points best: that is, $m = m'$ and $n = n'$



Original space where the data points are

Hough space defined by the parameters of the model we want to fit (i.e., m, n)

Hough transform

P.V.C. Hough, *Machine Analysis of Bubble Chamber Pictures,* Proc. Int. Conf. High Energy Accelerators and Instrumentation, 1959

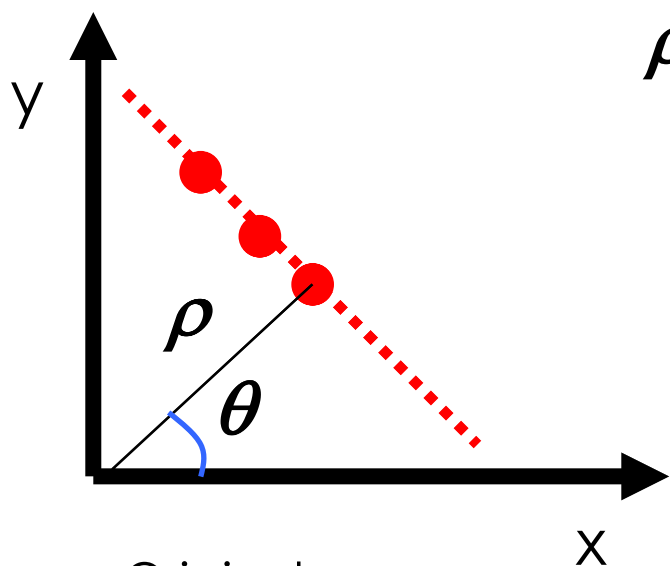
Any Issue? The parameter space [m,n] is unbounded…

Hough transform

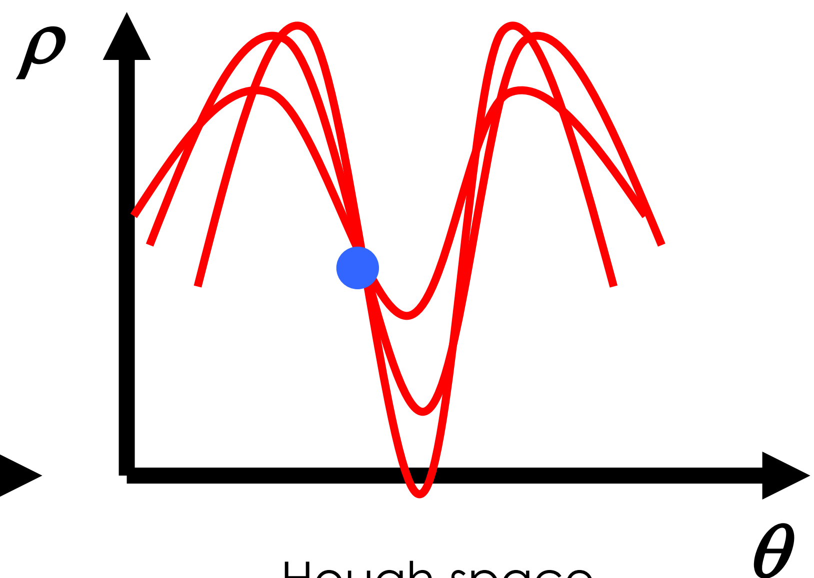
P.V.C. Hough, *Machine Analysis of Bubble Chamber Pictures,* Proc. Int. Conf. High Energy Accelerators and Instrumentation, 1959

Any Issue? The parameter space [m,n] is unbounded…

Use a polar representation for the parameter space



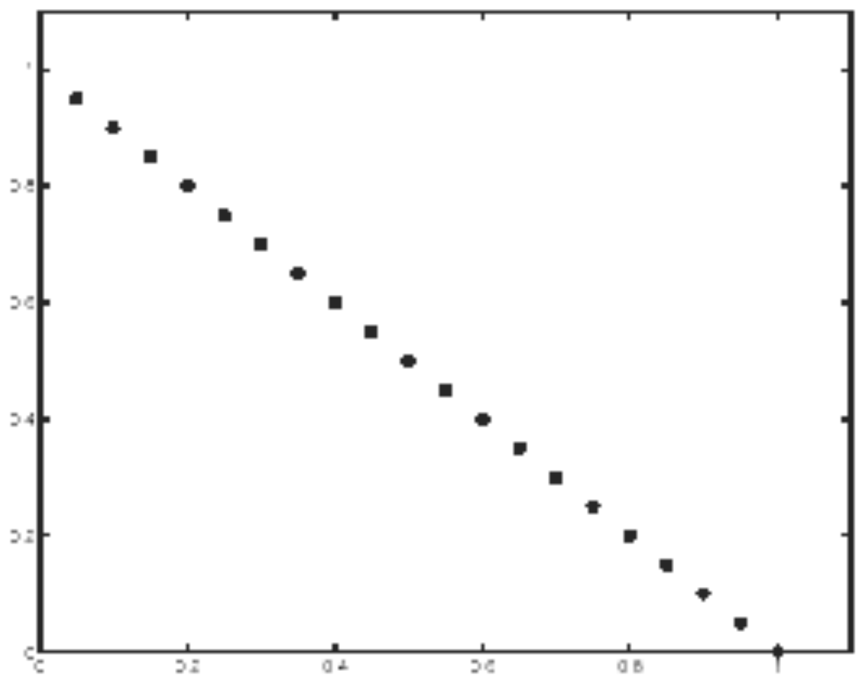
Original space



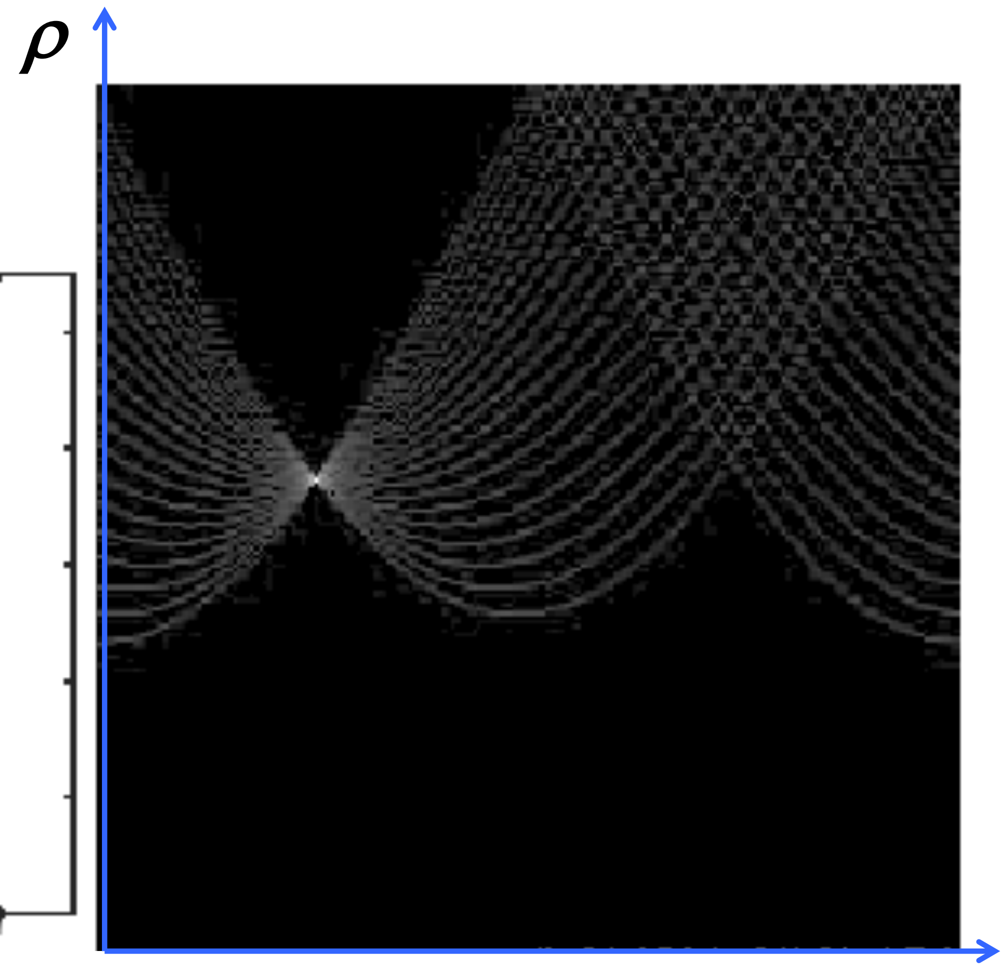
Hough space

$$x\cos\theta + y\sin\theta = \rho$$ [Eq. 13]

Hough transform - experiments



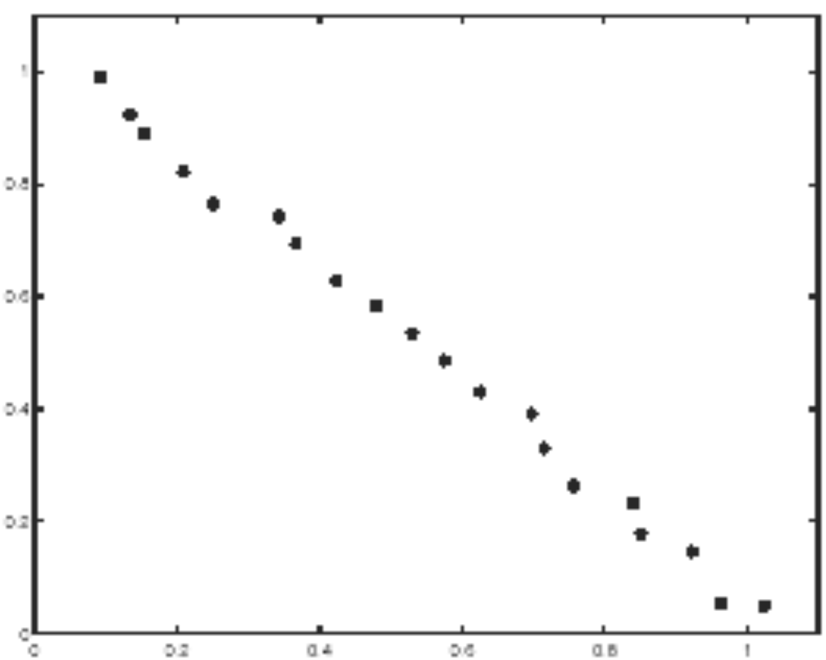
Original space



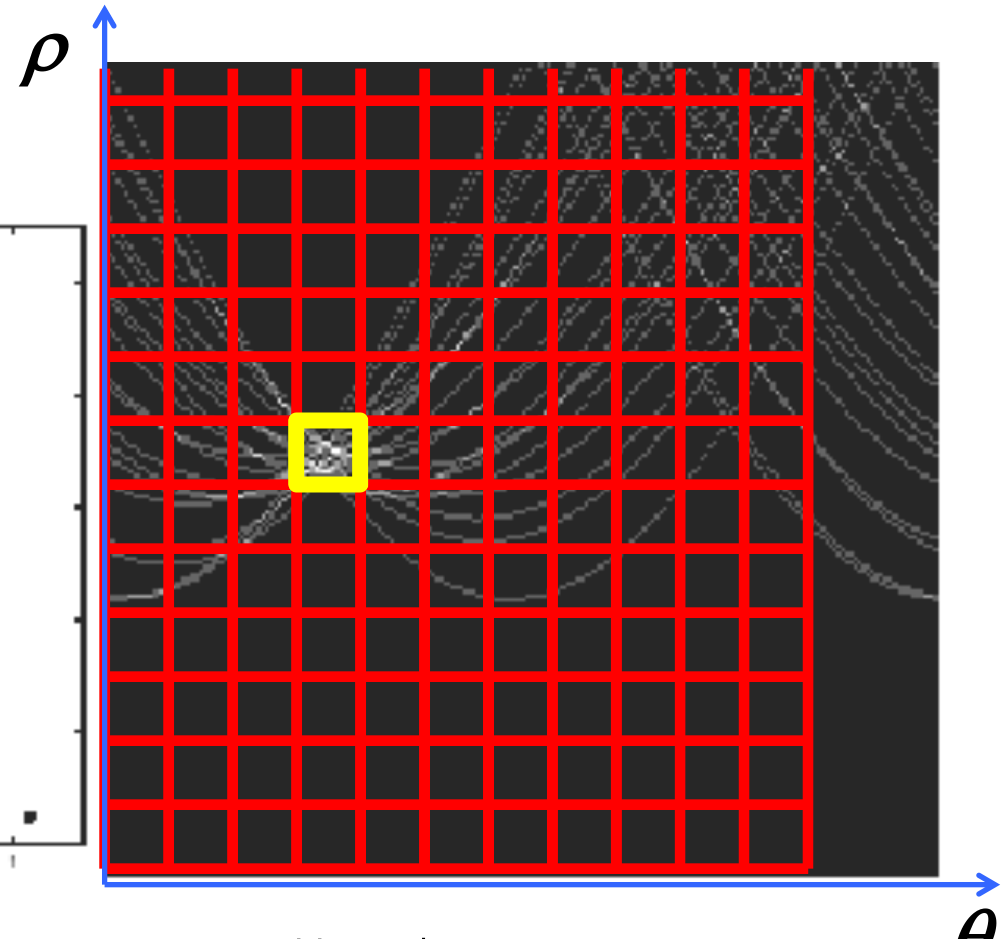
Hough space

Hough transform - experiments

Noisy data



Original space



Hough space

How to compute the intersection point? In presence of noise!
IDEA: introduce a grid a count intersection points in each cell

Issue: Grid size needs to be adjusted…

Hough transform - conclusions

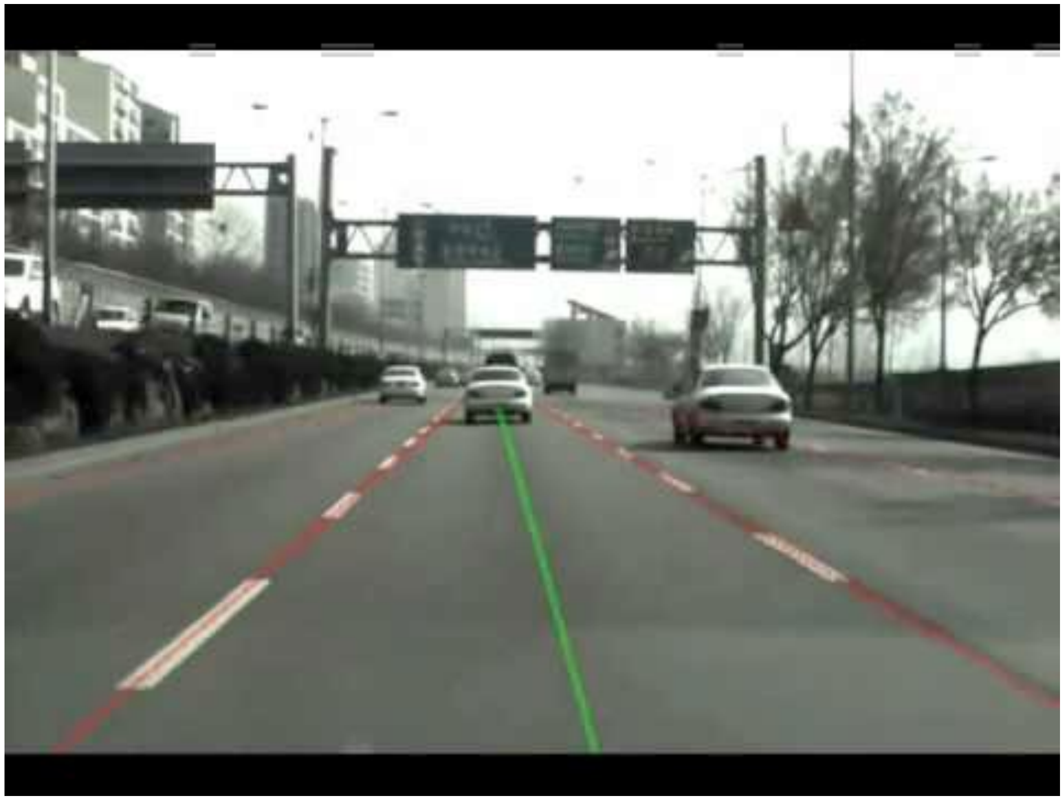
Good:

- All points are processed independently, so can cope with occlusion/outliers
- Some robustness to noise: noise points unlikely to contribute consistently to any single cell

Bad:

- Spurious peaks due to uniform noise
- Trade-off noise-grid size (hard to find sweet point)
- Doesn’t handle well high dimensional models

Applications – lane detection



Courtesy of Minchae Lee

Applications – computing vanishing points

Generalized Hough transform

[more on forthcoming lectures]

D. Ballard, Generalizing the Hough Transform to Detect Arbitrary Shapes, Pattern Recognition 13(2), 1981

- Parameterize a shape by measuring the location of its parts and shape centroid
- Given a set of measurements, cast a vote in the Hough (parameter) space

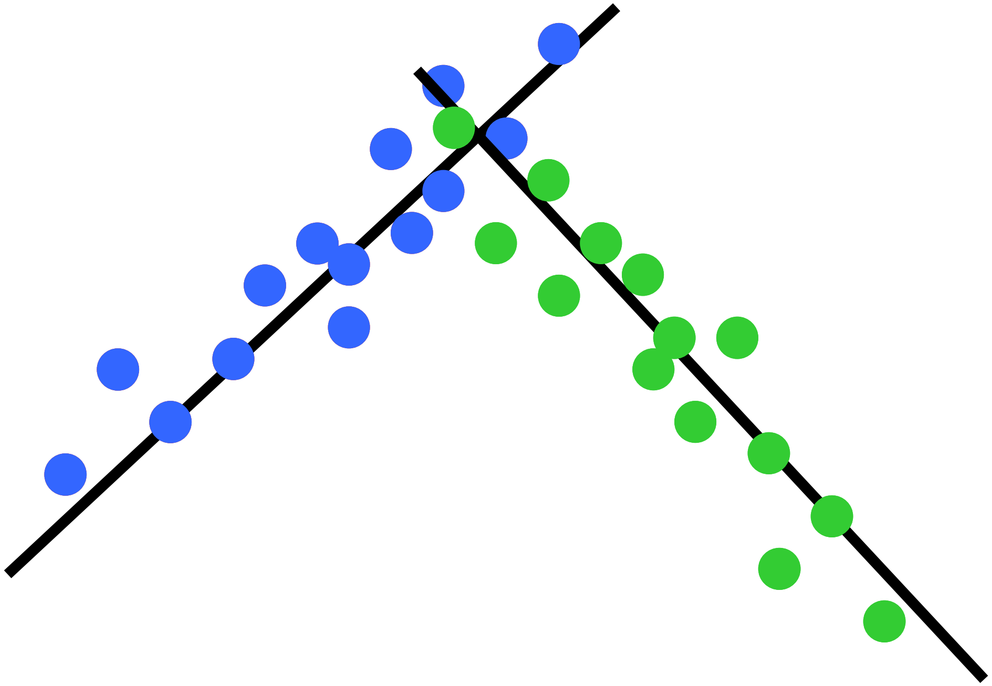
- Used in object recognition! (the implicit shape model)

B. Leibe, A. Leonardis, and B. Schiele, [Combined Object Categorization and Segmentation with an Implicit Shape Model](), ECCV Workshop on Statistical Learning in Computer Vision 2004

Lecture 9
Fitting and Matching

- Problem formulation
- Least square methods
- RANSAC
- Hough transforms
- Multi-model fitting
- Fitting helps matching!

Fitting multiple models



- Incremental fitting
- E.M. (probabilistic fitting)
- Hough transform

Incremental line fitting

Scan data point sequentially (using locality constraints)

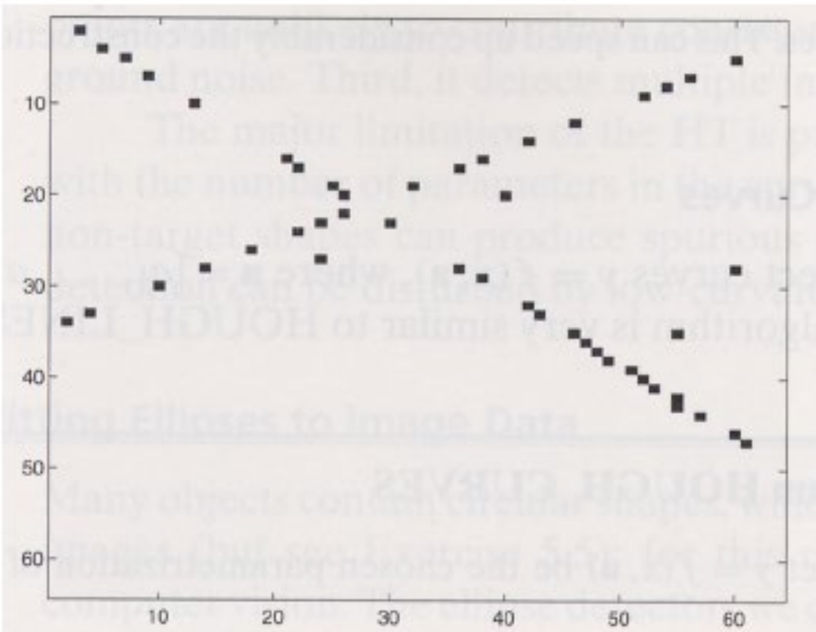
Perform following loop:

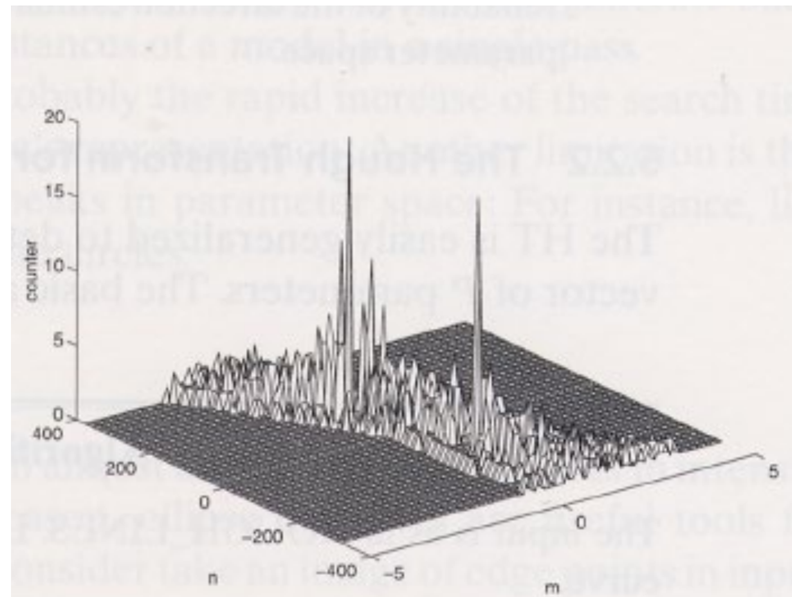
1. Select N point and fit line to N points
2. Compute residual R_N
3. Add a new point, re-fit line and re-compute R_{N+1}
4. Continue while line fitting residual is small enough,

➢ When residual exceeds a threshold, start fitting new model (line)

Hough transform

Courtesy of unknown



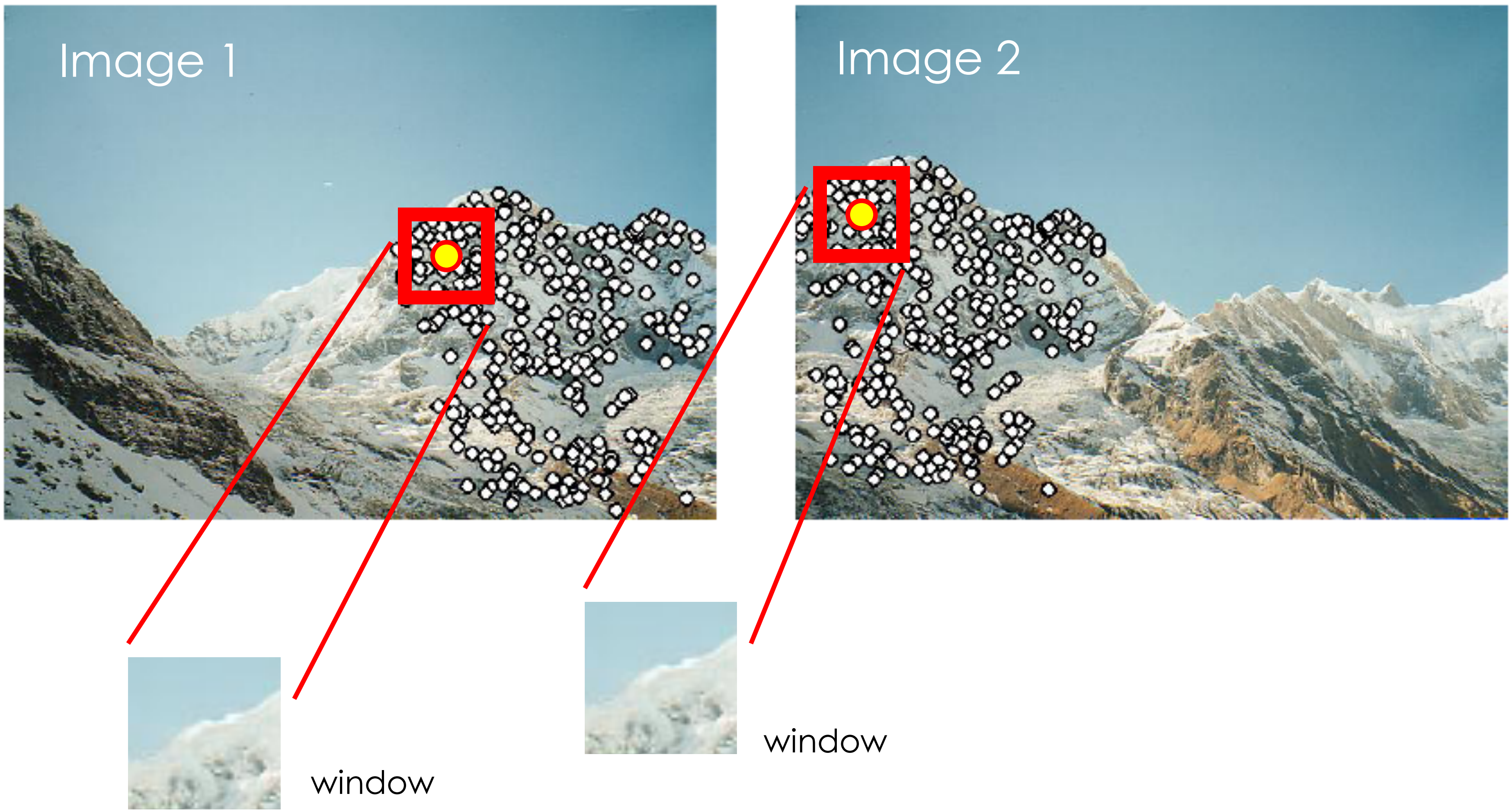


Same cons and pros as before…

Lecture 9
Fitting and Matching

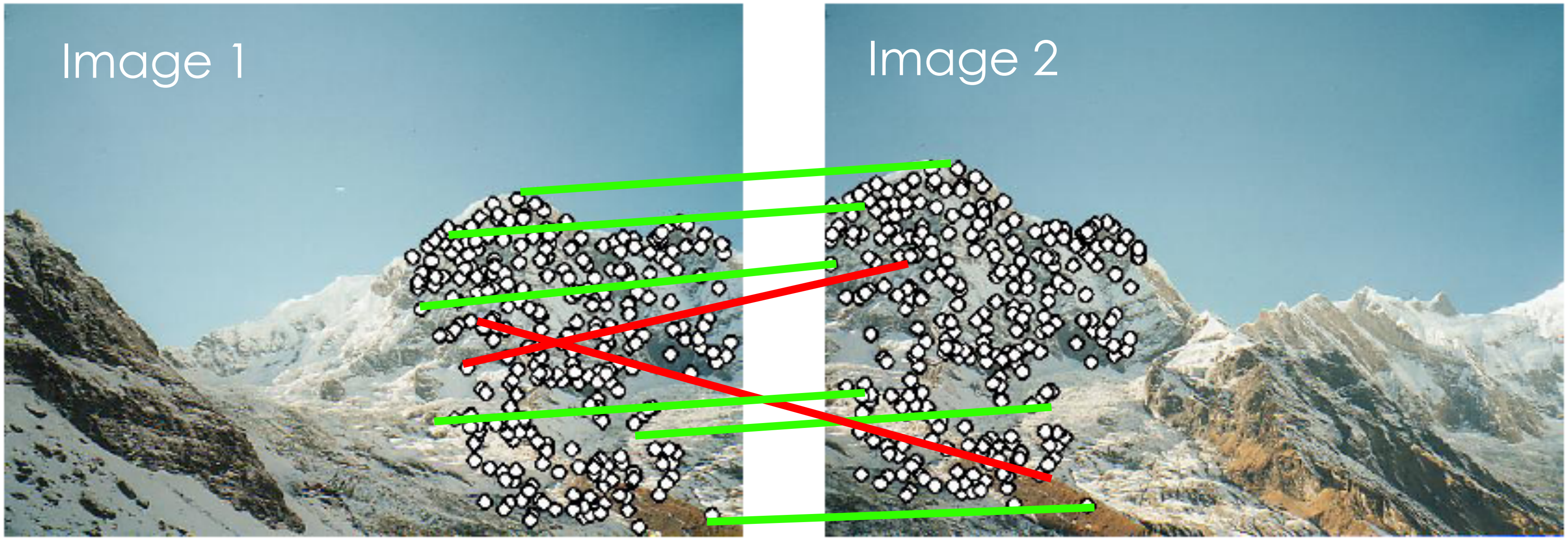
- Problem formulation
- Least square methods
- RANSAC
- Hough transforms
- Multi-model fitting
- Fitting helps matching!

Fitting helps matching!



Features are matched (for instance, based on correlation)

Fitting helps matching!



Matches based on appearance only
Green: good matches
Red: bad matches

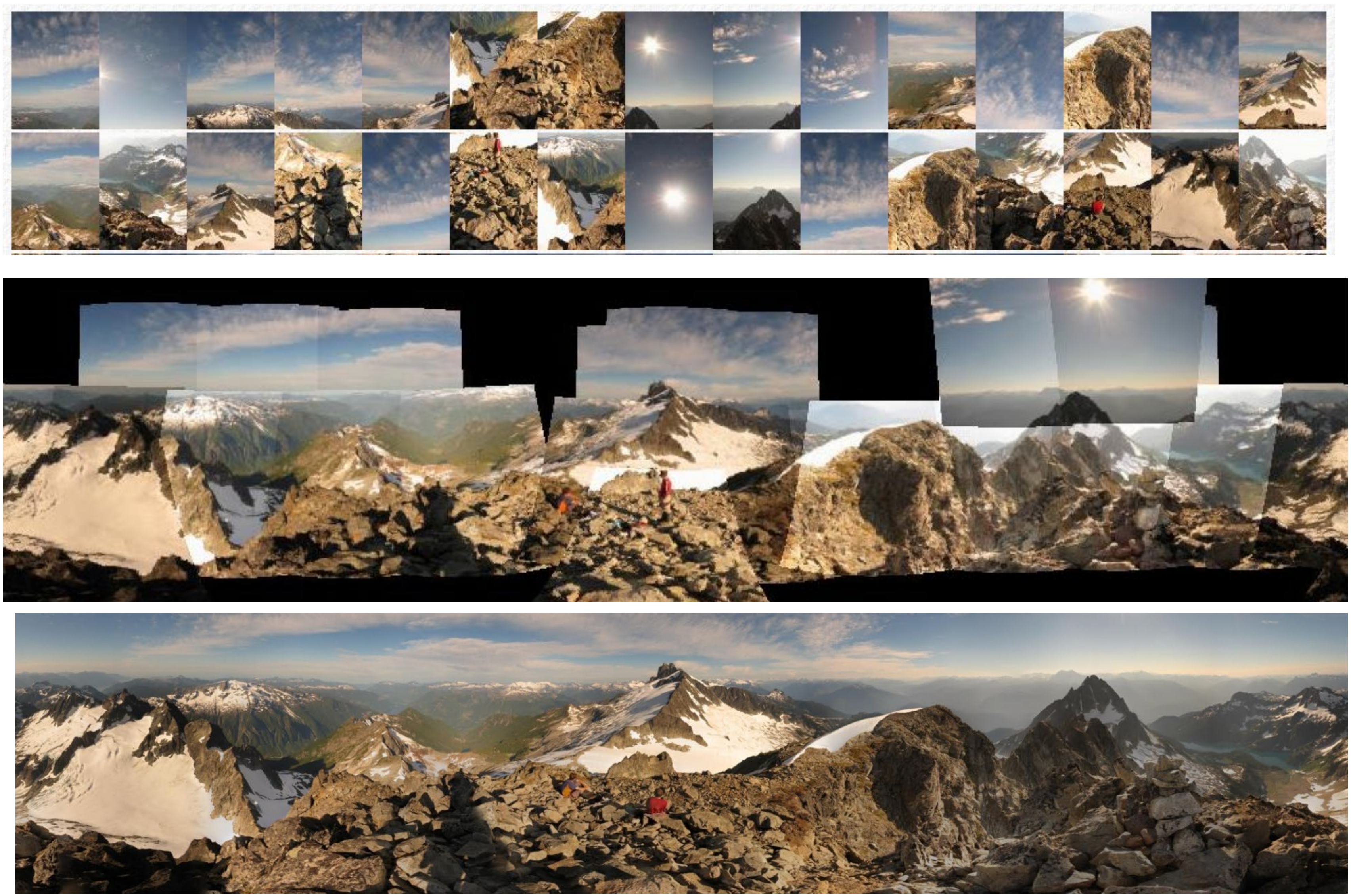
Idea:

- Fitting an homography H (by RANSAC) mapping features from images 1 to 2
- Bad matches will be labeled as outliers (hence rejected)!

Fitting helps matching!

Recognising Panoramas

M. Brown and D. G. Lowe. Recognising Panoramas. In Proceedings of the *9th International Conference on Computer Vision -- ICCV2003*



Next lecture:

Low Level Representations

Least squares methods

- fitting a line -

$$Ax = b$$

- More equations than unknowns
- Look for solution which minimizes $||Ax-b|| = (Ax-b)^T(Ax-b)$
- Solve $\dfrac{\partial (Ax-b)^T (Ax-b)}{\partial x_i} = 0$
- LS solution

$$x = (A^T A)^{-1} A^T b$$

Least squares methods
- fitting a line -

Solving $x = (A^t A)^{-1} A^t b$

$\mathrm{A}^+ = (\mathrm{A}^t\mathrm{A})^{-1}\mathrm{A}^t$ = pseudo-inverse of A

$\mathrm{A} = \mathrm{U}\sum\mathrm{V}^t$ = SVD decomposition of A

$A^{-1} = V\sum^{-1}U^T$

$A^+ = V\sum^+U^T$

with $\sum^+$ equal to $\sum^{-1}$ for all nonzero singular values and zero otherwise

Least squares methods

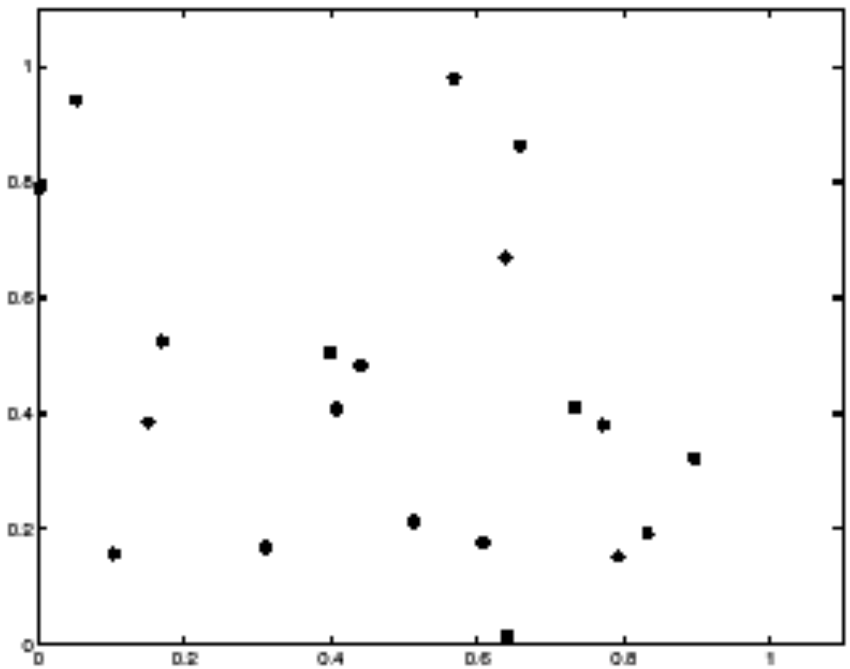
- fitting an homography -

$$
\begin{aligned}
h_{11}x + h_{12}y + h_{13} - h_{31}xx' - h_{32}yx' - x' &= 0 \\
h_{21}x + h_{22}y + h_{23} - h_{31}xy' - h_{32}yy' - y' &= 0
\end{aligned}
$$

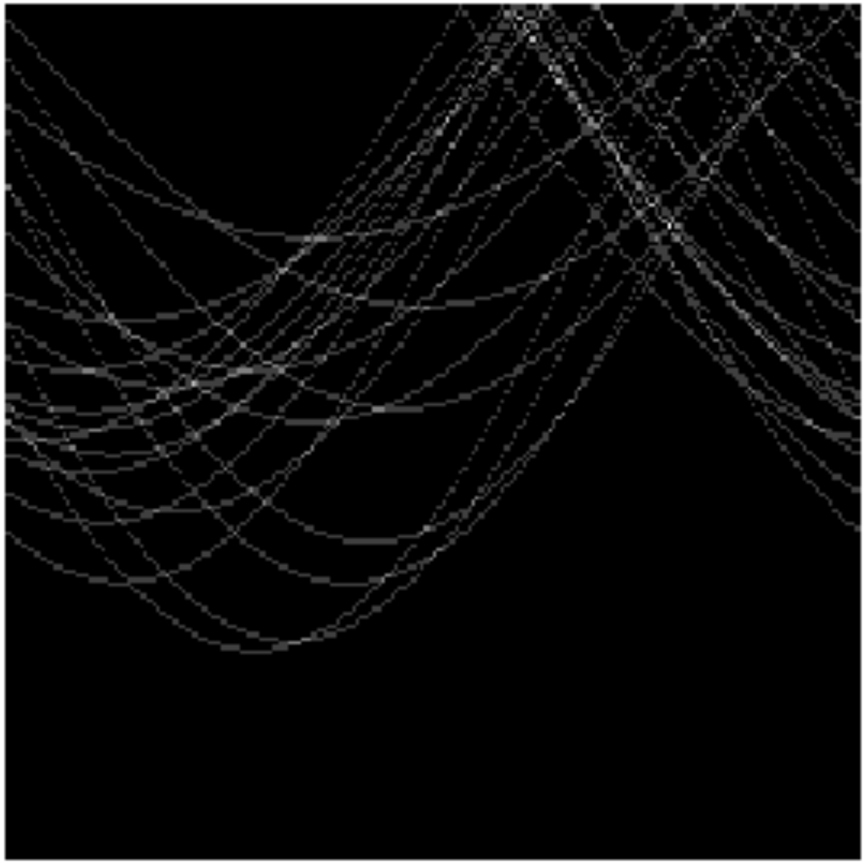
From n>=4 corresponding points: $\boxed{\mathrm{A}\,\mathrm{h} = 0}$

$$
\begin{pmatrix}
x_1 & y_1 & 1 & 0 & 0 & 0 & -x_1x_1' & -y_1x_1' & -x_1' \\
0 & 0 & 0 & x_1 & y_1 & 1 & -x_1y_1' & -y_1y_1' & -y_1' \\
x_2 & y_2 & 1 & 0 & 0 & 0 & -x_2x_2' & -y_2x_2' & -x_2' \\
0 & 0 & 0 & x_2 & y_2 & 1 & -x_2y_2' & -y_2y_2' & -y_2' \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
x_n & y_n & 1 & 0 & 0 & 0 & -x_nx_n' & -y_nx_n' & -x_n' \\
0 & 0 & 0 & x_n & y_n & 1 & -x_ny_n' & -y_ny_n' & -y_n'
\end{pmatrix}
\begin{bmatrix}
\mathrm{h}_{1,1} \\ \mathrm{h}_{1,2} \\ \vdots \\ \mathrm{h}_{3,3}
\end{bmatrix}
= 0
$$

Hough transform - experiments



features



votes

Issue: spurious peaks due to uniform noise