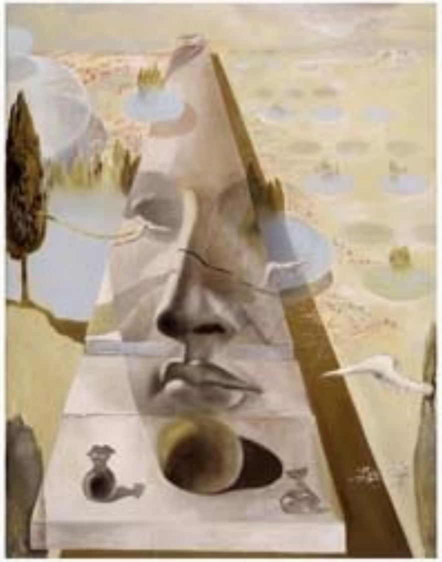
Lecture 4
Single View Metrology



Professor Silvio Savarese

Computational Vision and Geometry Lab

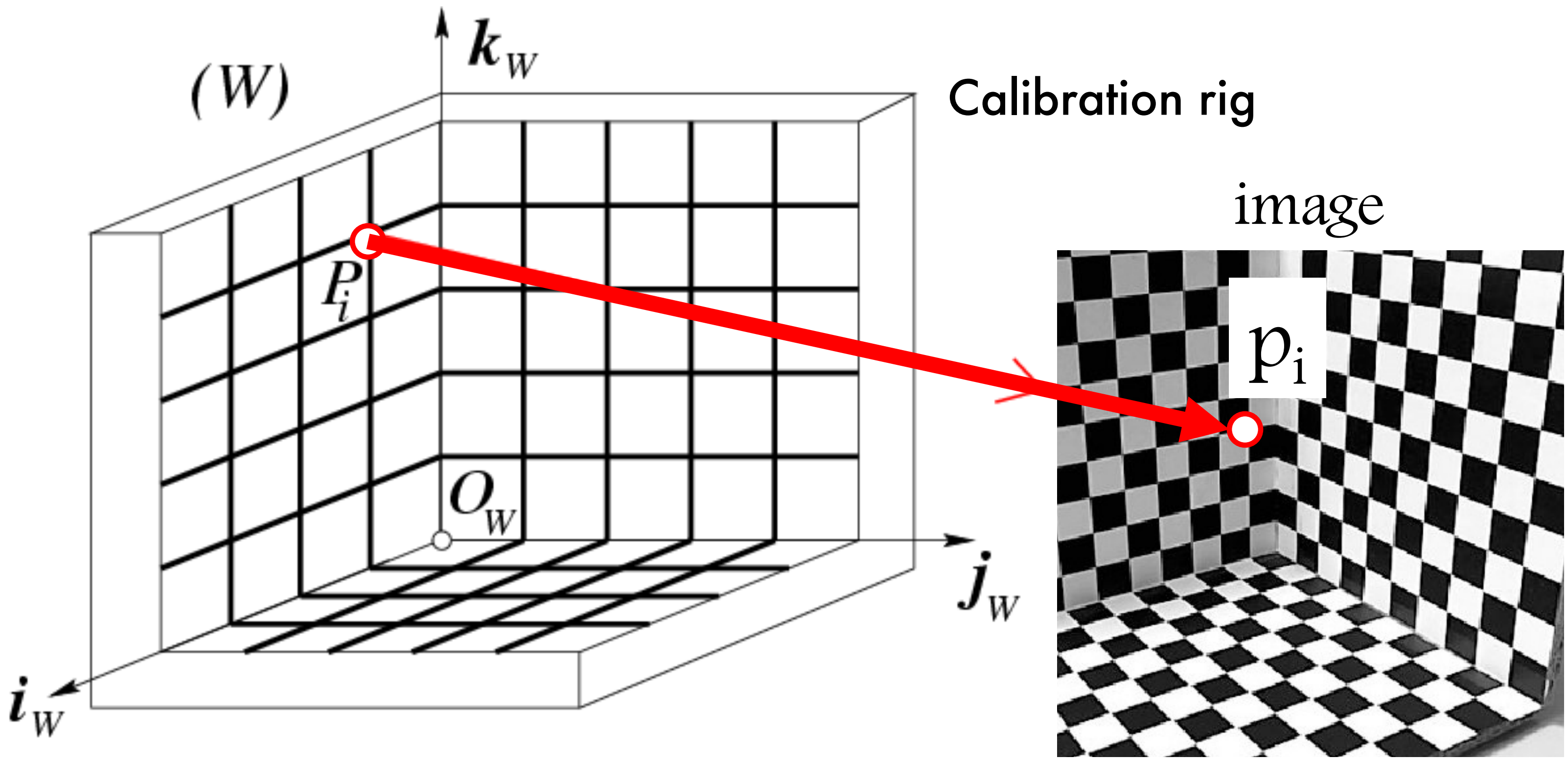
Lecture 4

Single View Metrology

- Review calibration and 2D transformations
- Vanishing points and lines
- Estimating geometry from a single image
- Extensions

Reading:
[HZ] Chapter 2 “Projective Geometry and Transformation in 2D”
[HZ] Chapter 3 “Projective Geometry and Transformation in 3D”
[HZ] Chapter 8 “More Single View Geometry”
[Hoeim & Savarese] Chapter 2

Calibration Problem



$$p_i = \begin{bmatrix} u_i \\ v_i \end{bmatrix} = M \; P_i$$

In pixels

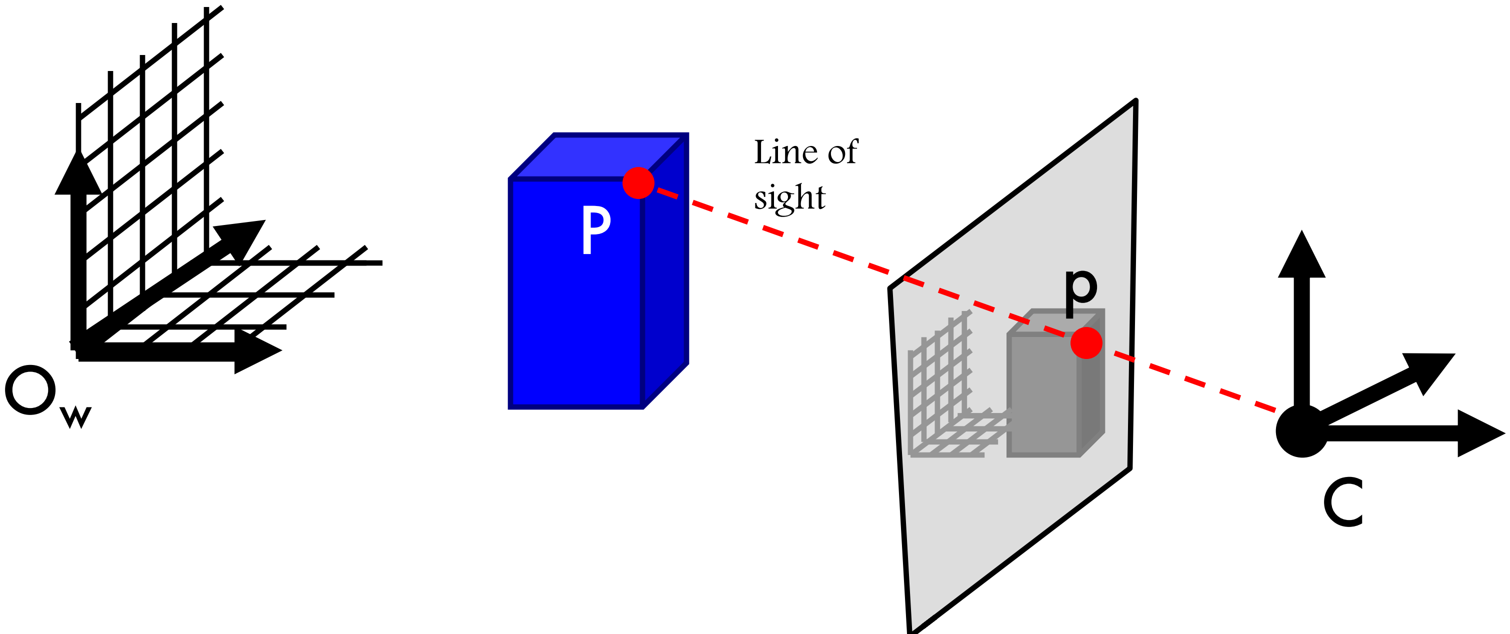
World ref. system

$$M = K\begin{bmatrix} R & T \end{bmatrix}$$

11 unknowns

Need at least 6 correspondences

Once the camera is calibrated...



$$M = K[R \quad T]$$

-Internal parameters K are known

-R, T are known – but these can only relate C to the calibration rig

Can I estimate P from the measurement p from a single image?

No - in general ☹ (P can be anywhere along the line defined by C and p)

Recovering structure from a single view

http://www.robots.ox.ac.uk/~vgg/projects/SingleView/models/hut/hutme.wrl

Transformation in 2D

-Isometries

-Similarities

-Affinity

-Projective

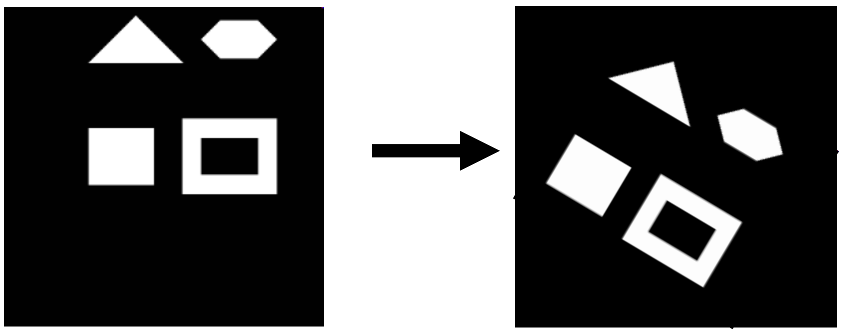
Transformation in 2D

Isometries:

[Euclideans]

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} R & t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = H_e \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$ [Eq. 4]

- Preserve distance (areas)
- 3 DOF
- Regulate motion of rigid object



Transformation in 2D

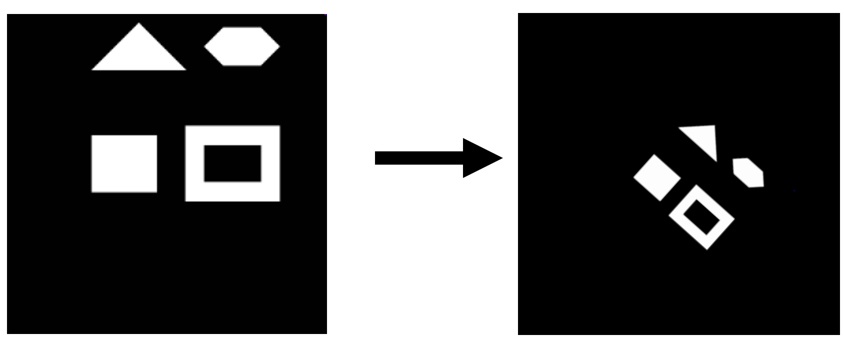
Similarities:

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} S\,R & t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = H_s \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$S = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix}$$

[Eq. 5]

- Preserve
    - ratio of lengths
    - angles
-4 DOF



Transformation in 2D

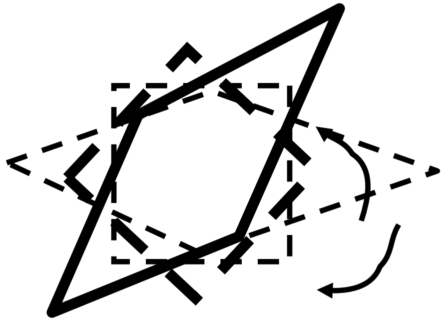
Affinities:

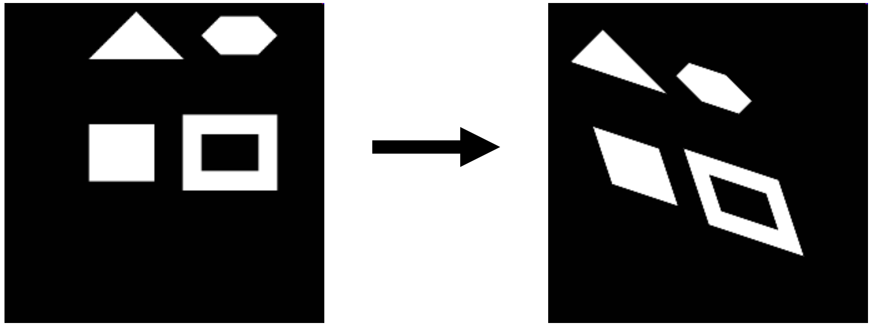
$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} A & t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = H_a \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

[Eq. 6]

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = R(\boldsymbol{\theta}) \cdot R(-\boldsymbol{\phi}) \cdot D \cdot R(\boldsymbol{\phi}) \qquad D = \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix}$$

[Eq. 7]





Transformation in 2D

Affinities:

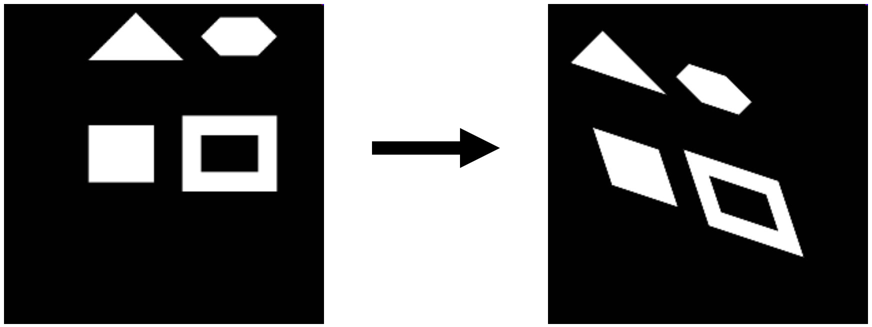
$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} A & t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = H_a \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$ [Eq. 6]

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = R(\theta) \cdot R(-\phi) \cdot D \cdot R(\phi) \qquad D = \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix}$$

[Eq. 7]

-Preserve:
- Parallel lines
- Ratio of areas
- Ratio of lengths on collinear lines
- others…

- 6 DOF



Transformation in 2D

Affinities:

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} A & t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = H_a \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

[Eq. 6]

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = R(\theta) \cdot R(-\phi) \cdot D \cdot R(\phi) \qquad D = \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix}$$

[Eq. 7]

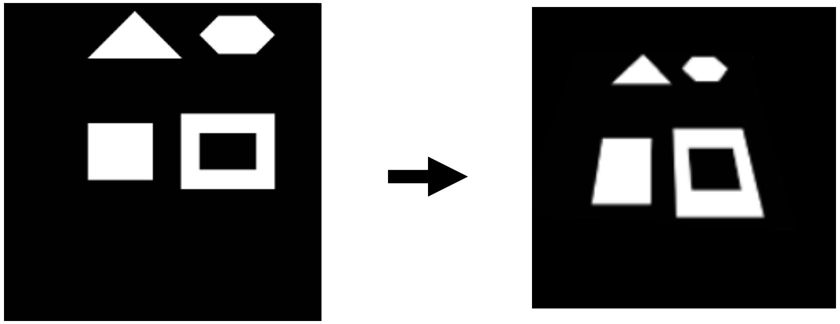
$$A = UDV^T = UV^T VDV^T = (UV^T)\,(V\,)(D)\,(V^T)$$

Transformation in 2D

Projective:

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} A & t \\ v & b \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = H_p \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$ [Eq. 8]

- 8 DOF
- Preserve:
    - collinearity
    - and a few others…



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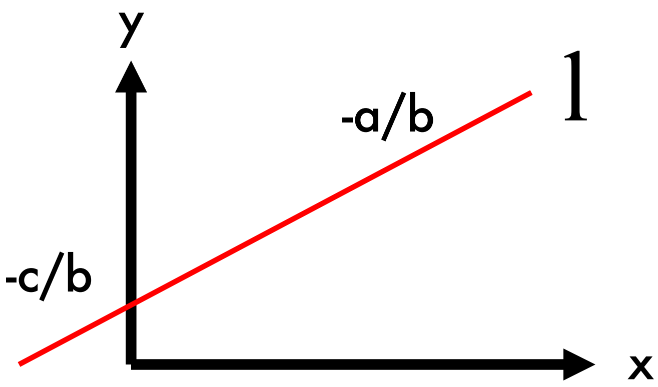
Lines in a 2D plane

$$ax + by + c = 0$$



$$\mathbf{l} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

If $\mathbf{x} = [\, x_1, x_2]^T \in \mathbf{l}$

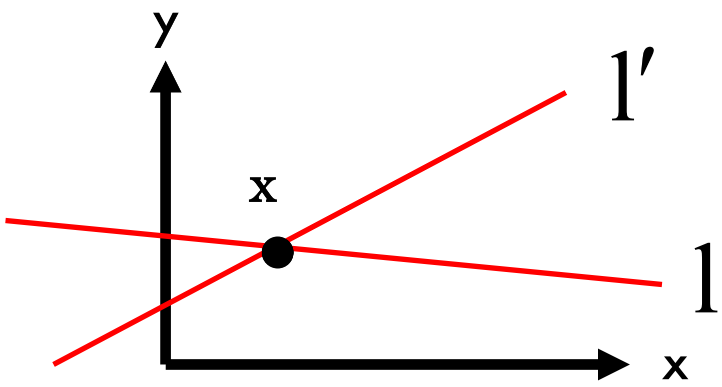
$$\begin{bmatrix} x_1 \\ x_2 \\ 1 \end{bmatrix}^T \begin{bmatrix} a \\ b \\ c \end{bmatrix} = 0$$

[Eq. 10]

Lines in a 2D plane

Intersecting lines

$$\mathrm{x} = \mathrm{l} \times \mathrm{l}' \quad \text{[Eq. 11]}$$



Proof

$$\mathrm{l} \times \mathrm{l}' \perp \mathrm{l} \quad \rightarrow (\mathrm{l} \times \mathrm{l}') \cdot \mathrm{l} = 0 \quad \rightarrow x \in l \quad \text{[Eq. 12]}$$

$$\mathrm{l} \times \mathrm{l}' \perp \mathrm{l}' \quad \rightarrow \underbrace{(\mathrm{l} \times \mathrm{l}')}_{\mathrm{x}} \cdot \mathrm{l}' = 0 \quad \rightarrow x \in l' \quad \text{[Eq. 13]}$$

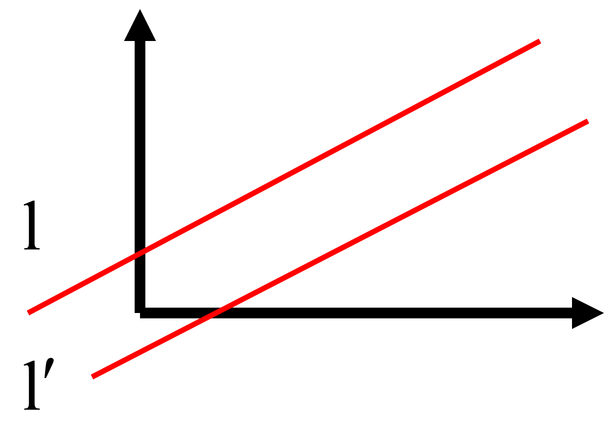
$\rightarrow$ x is the intersection point

2D Points at infinity (ideal points)

$$\mathrm{x} = \begin{bmatrix} \mathrm{x}_1 \\ \mathrm{x}_2 \\ \mathrm{x}_3 \end{bmatrix}, \mathrm{x}_3 \neq 0$$

$$x_\infty = \begin{bmatrix} x'_1 \\ x'_2 \\ 0 \end{bmatrix}$$

$$-a/b = -a'/b'$$



$$\mathrm{l} = \begin{bmatrix} \mathrm{a} \\ \mathrm{b} \\ \mathrm{c} \end{bmatrix}$$

$$l' = \begin{bmatrix} a' \\ b' \\ c' \end{bmatrix}$$

Let's intersect two parallel lines:

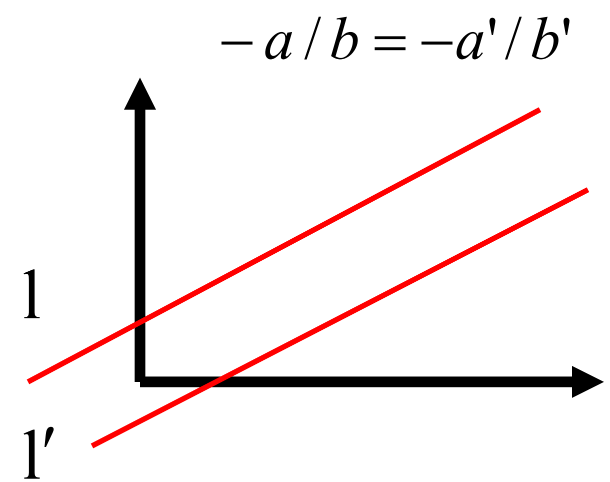
$$\rightarrow l \times l' \propto \begin{bmatrix} b \\ -a \\ 0 \end{bmatrix} = x_\infty \quad \text{[Eq.13]}$$

= ideal point!

- In Euclidian coordinates this point is at infinity
- Agree with the general idea of two lines intersecting at infinity

2D Points at infinity (ideal points)

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, x_3 \neq 0$$



$$\mathbf{l} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

$$l' = \begin{bmatrix} a' \\ b' \\ c' \end{bmatrix}$$

Note: the line l = [a b c]^T pass trough the ideal point x_∞ = [b -a 0]^T

$$\mathbf{l}^T x_\infty = \begin{bmatrix} a & b & c \end{bmatrix} \begin{bmatrix} b \\ -a \\ 0 \end{bmatrix} = 0 \quad \text{[Eq. 15]}$$

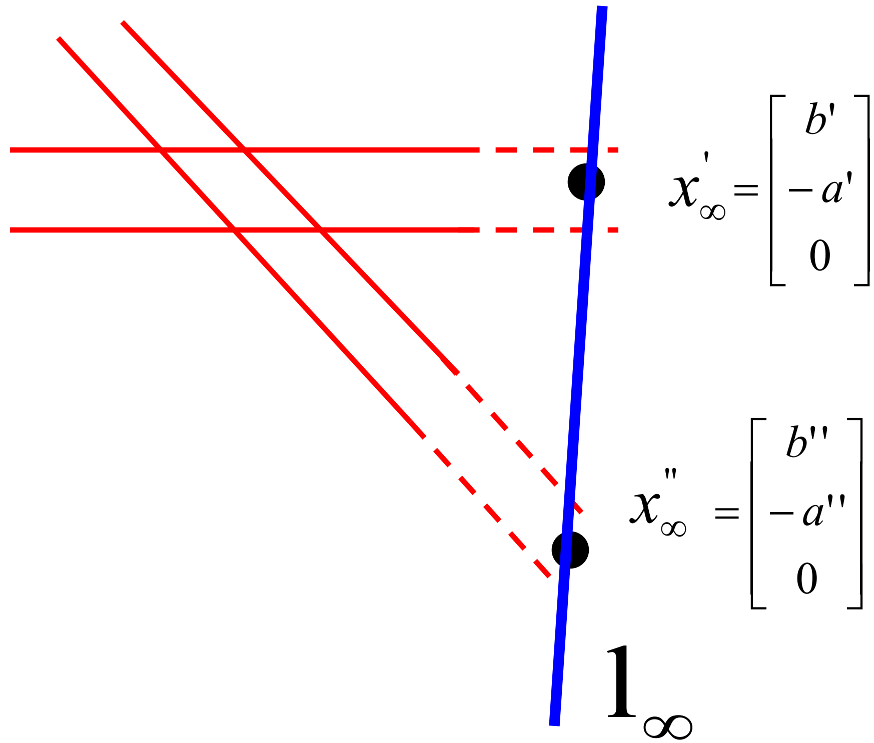
So does the line l′ since a b′ = a′ b

Lines infinity $\mathbf{l}_\infty$

Set of ideal points lies on a line called the line at infinity.
How does it look like?

$$\mathbf{l}_\infty = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

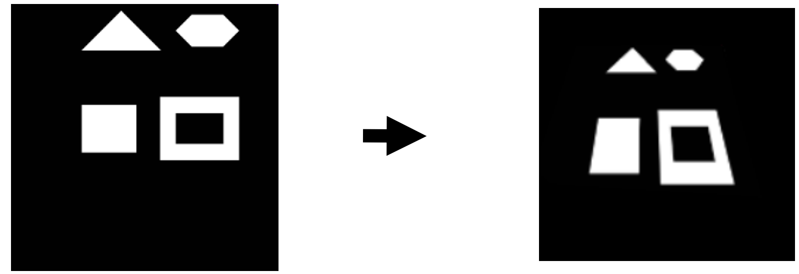
Indeed: $\begin{bmatrix} x_1 \\ x_2 \\ 0 \end{bmatrix}^T \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = 0$



A line at infinity can be thought of the set of "directions" of lines in the plane

Projective transformation of a point at infinity

$$H = \begin{bmatrix} A & t \\ v & b \end{bmatrix}$$



$$p' = H\ p$$

is it a point at infinity?

$$H\ p_\infty = ? = \begin{bmatrix} A & t \\ v & b \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} p'_x \\ p'_y \\ p'_z \end{bmatrix}$$

...no!

[Eq. 17]

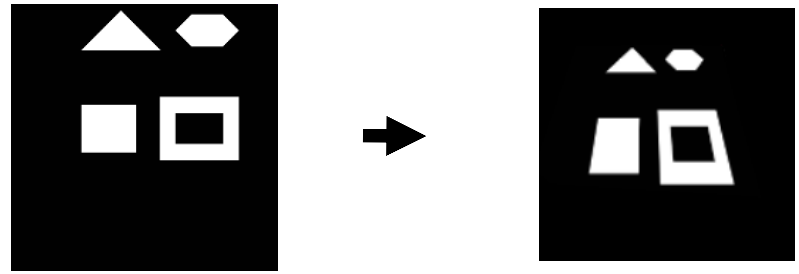
$$H_A\ p_\infty = ? = \begin{bmatrix} A & t \\ 0 & b \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} p'_x \\ p'_y \\ 0 \end{bmatrix}$$

An affine transformation of a point at infinity is still a point at infinity

[Eq. 18]

Projective transformation of a line (in 2D)

$$\mathrm{H} = \begin{bmatrix} \mathrm{A} & \mathrm{t} \\ \mathrm{v} & \mathrm{b} \end{bmatrix}$$



$$\mathrm{l}' = \mathrm{H}^{-\mathrm{T}}\ \mathrm{l}$$

[Eq. 19]

is it a line at infinity?

$$\mathrm{H}^{-\mathrm{T}}\ \mathrm{l}_{\infty} = ? = \begin{bmatrix} \mathrm{A} & \mathrm{t} \\ \mathrm{v} & \mathrm{b} \end{bmatrix}^{-\mathrm{T}} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \mathrm{t}_{\mathrm{x}} \\ \mathrm{t}_{\mathrm{y}} \\ \mathrm{b} \end{bmatrix}$$

...no!

[Eq. 20]

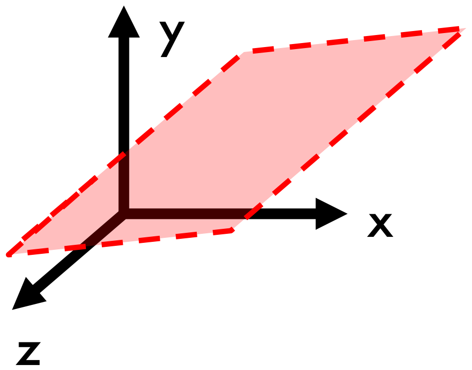
$$\mathrm{H}_{\mathrm{A}}^{-\mathrm{T}}\ \mathrm{l}_{\infty} = ? \quad = \begin{bmatrix} A & t \\ 0 & 1 \end{bmatrix}^{-T} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} A^{-T} & 0 \\ -t^{T} A^{-T} & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

[Eq. 21]

Points and planes in 3D

$$\mathrm{x} = \begin{bmatrix} \mathrm{x}_1 \\ \mathrm{x}_2 \\ \mathrm{x}_3 \\ 1 \end{bmatrix}$$

$$\Pi = \begin{bmatrix} \mathrm{a} \\ \mathrm{b} \\ \mathrm{c} \\ \mathrm{d} \end{bmatrix}$$



$$x \in \Pi \leftrightarrow x^T \Pi = 0$$

[Eq. 22]

$$\mathrm{ax} + \mathrm{by} + \mathrm{cz} + \mathrm{d} = 0$$

[Eq. 23]

Lines in 3D

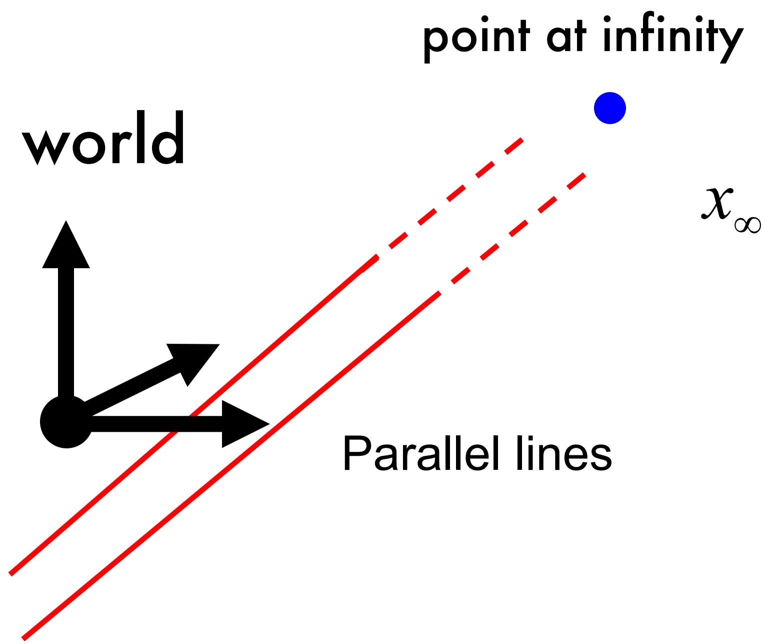
- Lines have 4 degrees of freedom - hard to represent in 3D-space

- Can be defined as intersection of 2 planes

$$\begin{aligned} \mathbf{d} &= \text{direction of the line} \\ &= [a, b, c]^T \end{aligned}$$

Points at infinity in 3D

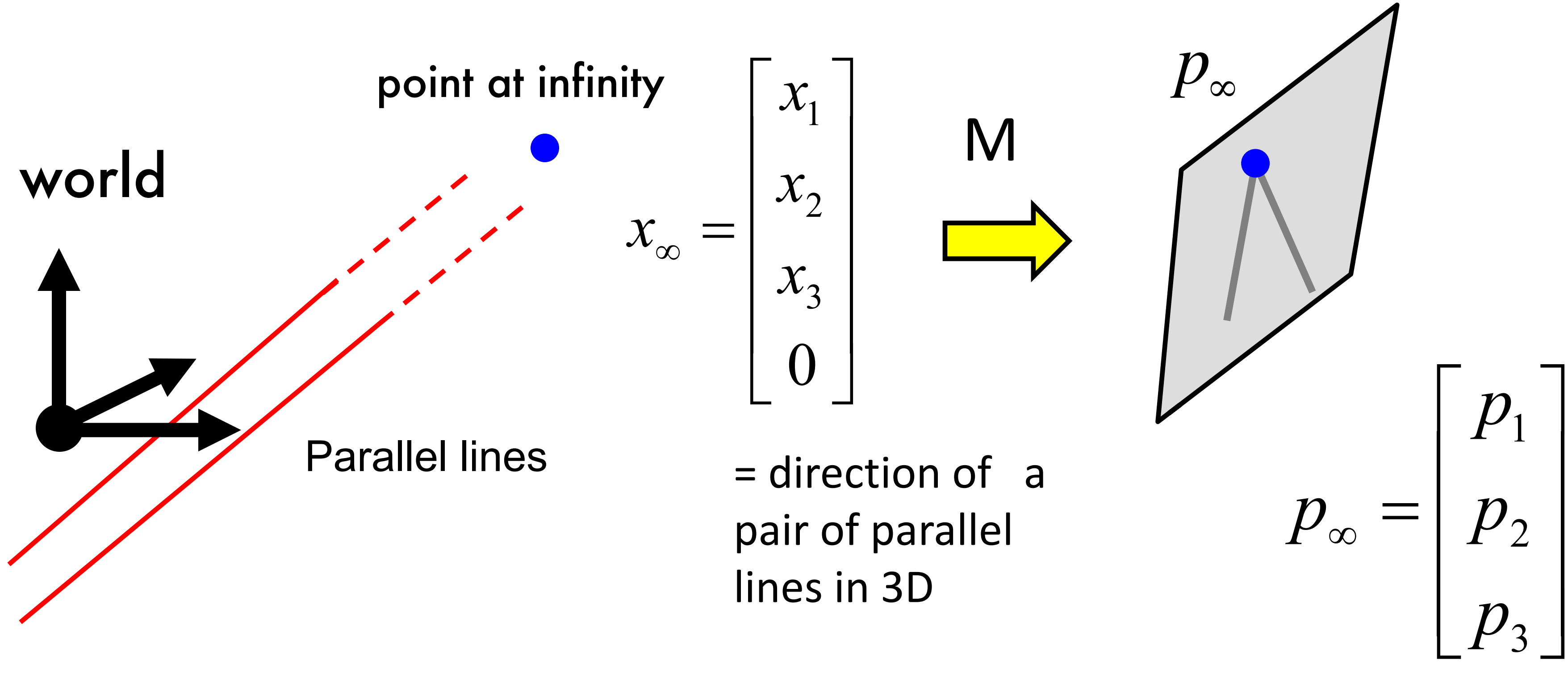
Points where parallel lines intersect in 3D



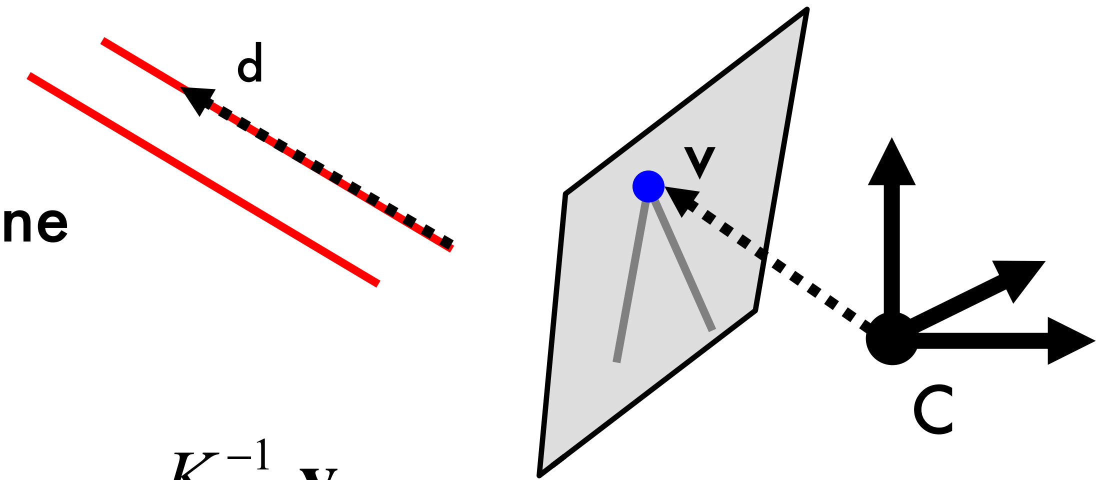
$$x_\infty = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ 0 \end{bmatrix}$$

Vanishing points

The projective projection of a point at infinity into the image plane defines a vanishing point.



Vanishing points and directions



d = direction of the line
= [a, b, c]^T

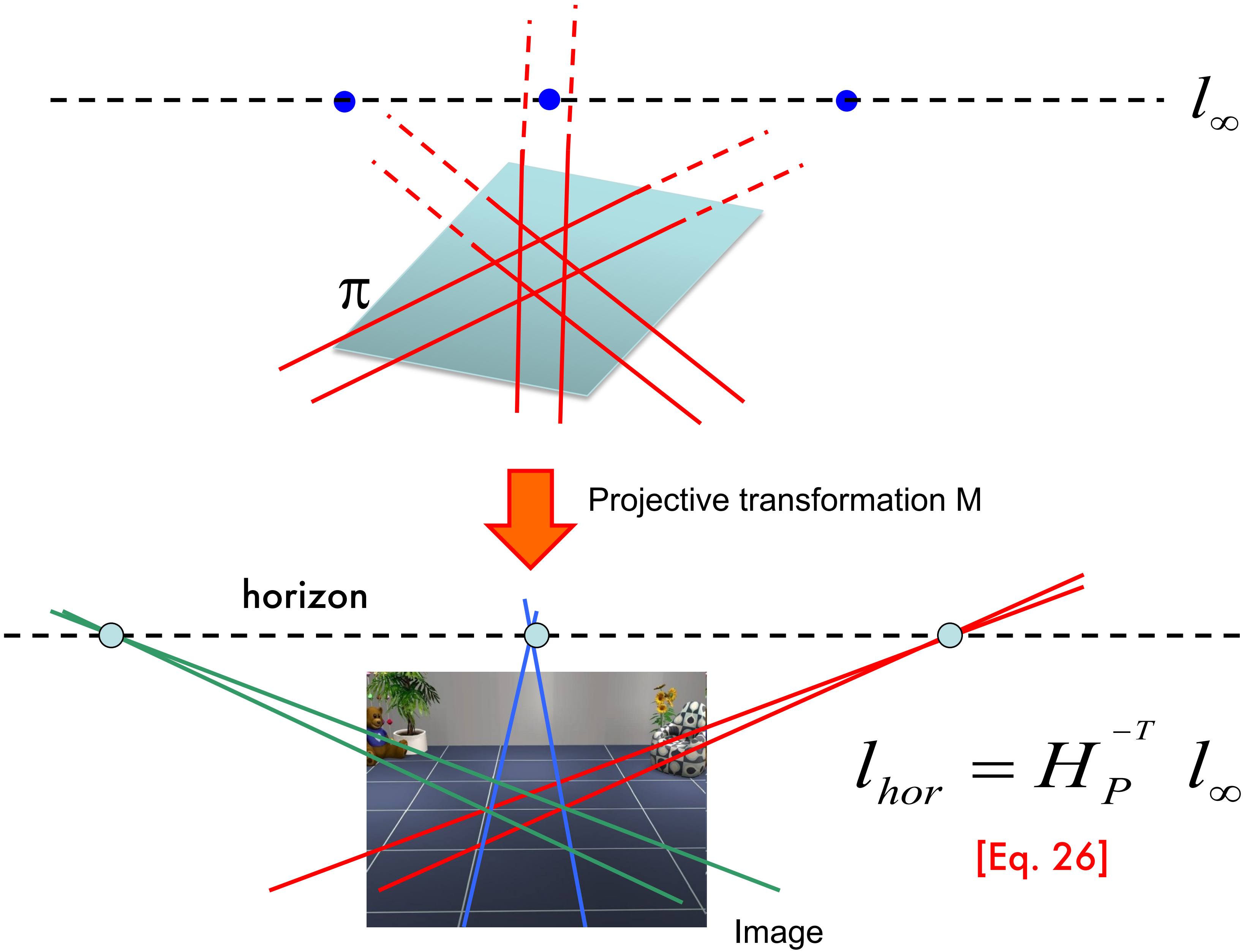
$$\mathbf{v} = K\,\mathbf{d}$$
[Eq. 24]

$$\mathbf{d} = \frac{K^{-1}\,\mathbf{v}}{\left\| K^{-1}\,\mathbf{v} \right\|}$$
[Eq. 25]

Proof:

$$X_\infty = \begin{bmatrix} a \\ b \\ c \\ 0 \end{bmatrix} \xrightarrow{M} \mathbf{v} = \mathbf{M}\,\mathbf{X}_\infty = \mathbf{K}\begin{bmatrix}\mathbf{I} & \mathbf{0}\end{bmatrix}\begin{bmatrix} a \\ b \\ c \\ 0 \end{bmatrix} = \mathbf{K}\begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

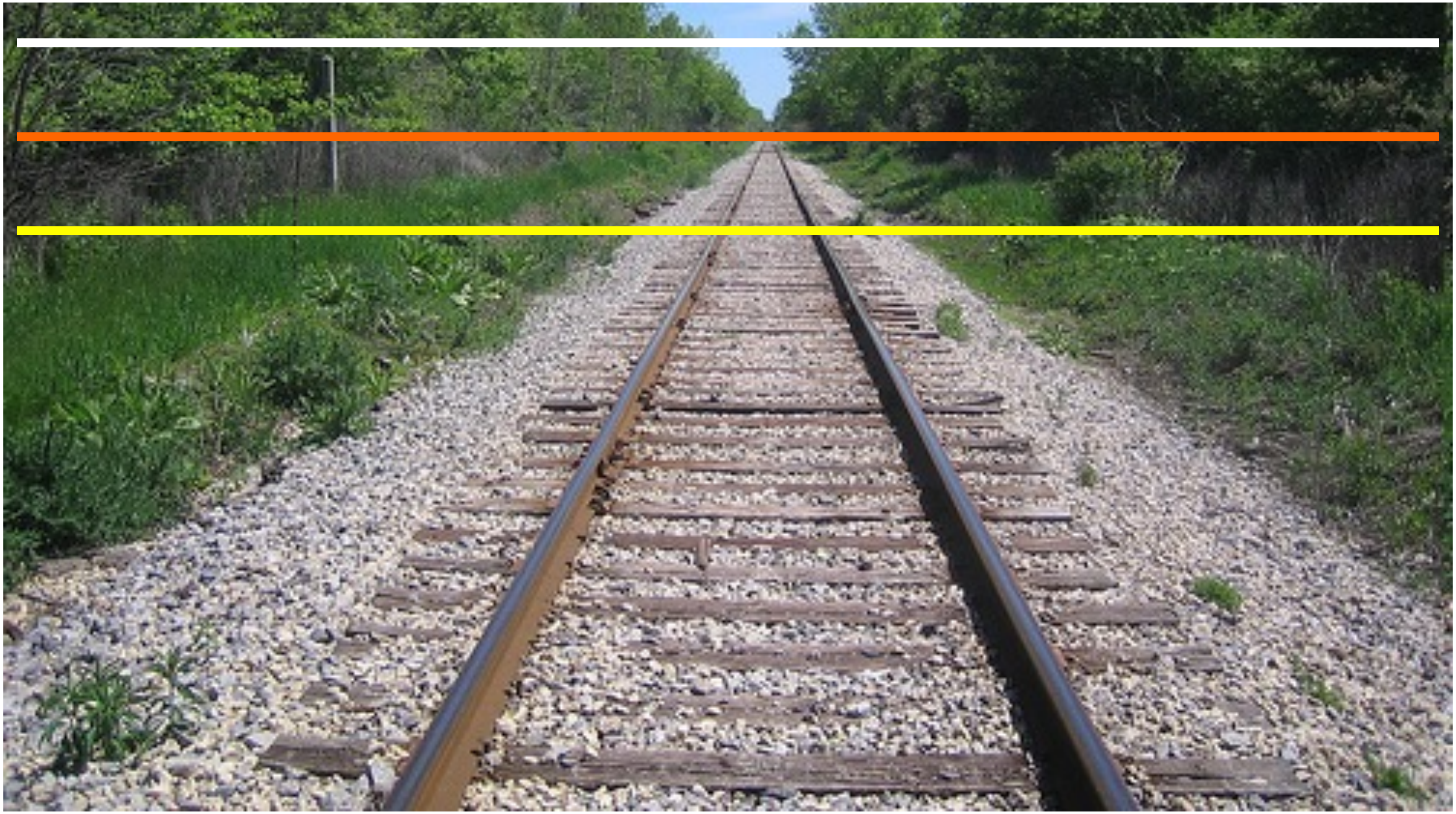
Vanishing (horizon) line



$$l_{hor} = H_P^{-T}\; l_\infty$$

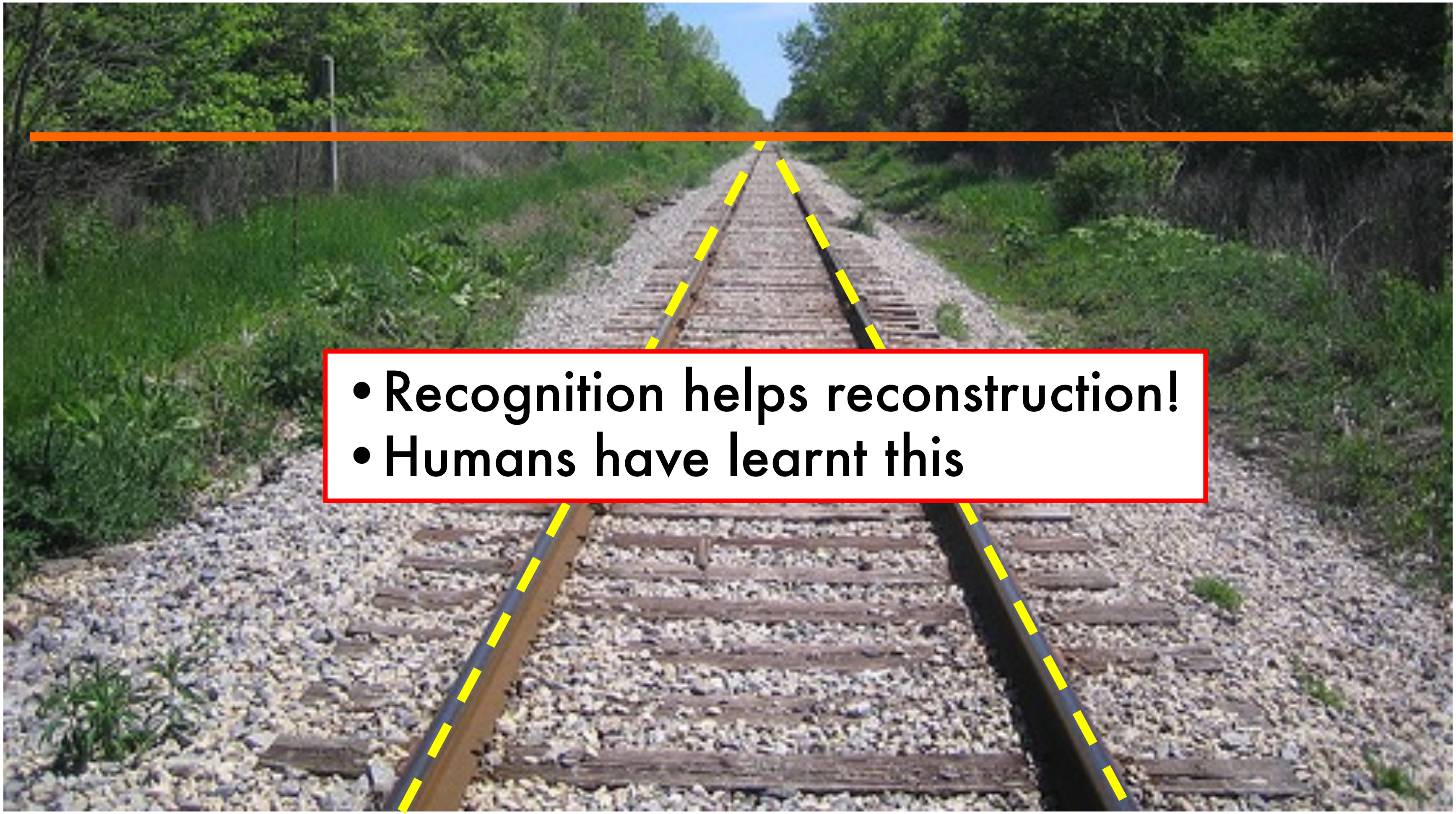
[Eq. 26]

Example of horizon line



The orange line is the horizon!

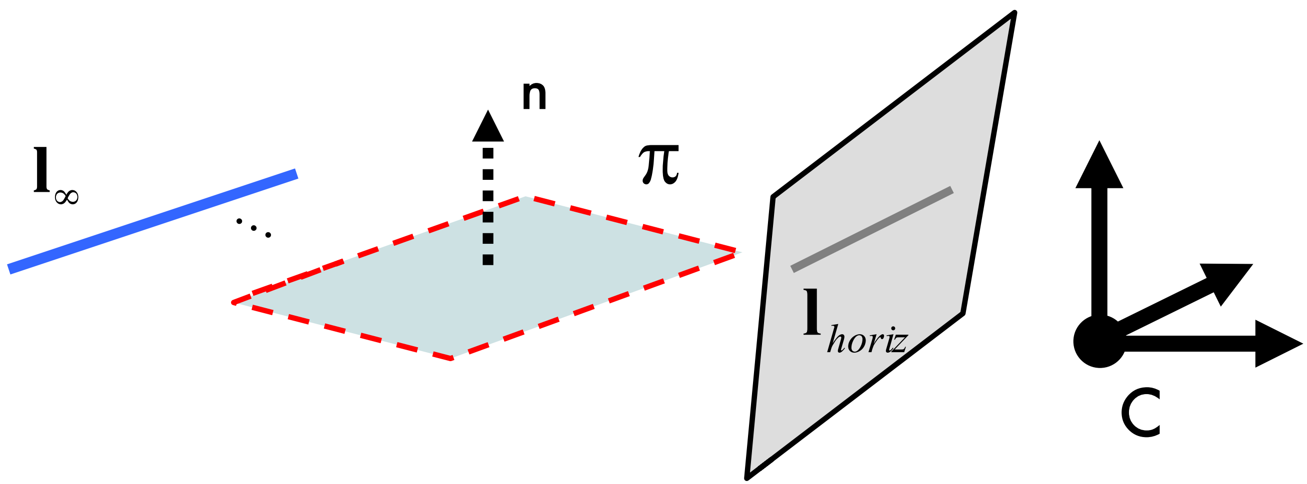
Are these two lines parallel or not?



- Recognition helps reconstruction!
- Humans have learnt this

- Recognize the horizon line
- Measure if the 2 lines meet at the horizon
- if yes, these 2 lines are // in 3D

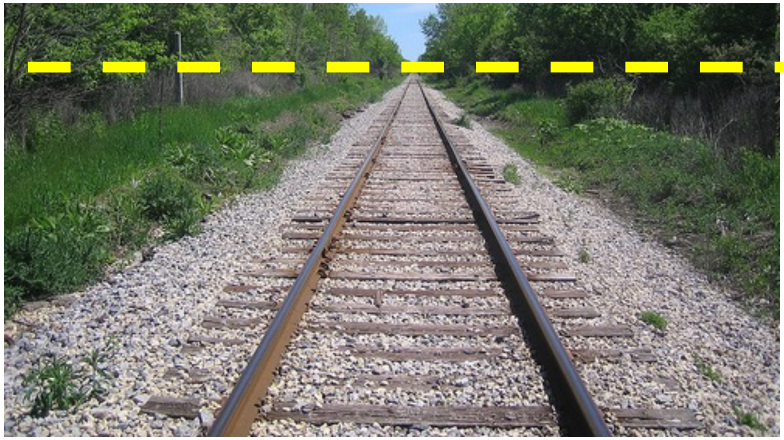
Vanishing points and planes



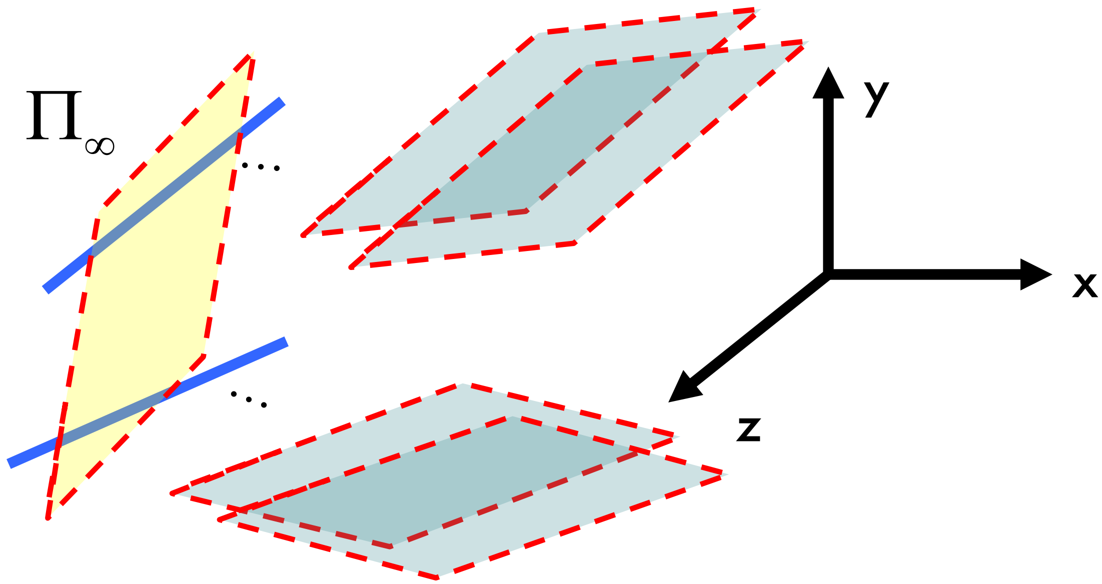
$$\mathbf{n} = K^T \mathbf{l}_{\text{horiz}}$$

[Eq. 27]

See sec. 8.6.2 [HZ] for details



Planes at infinity

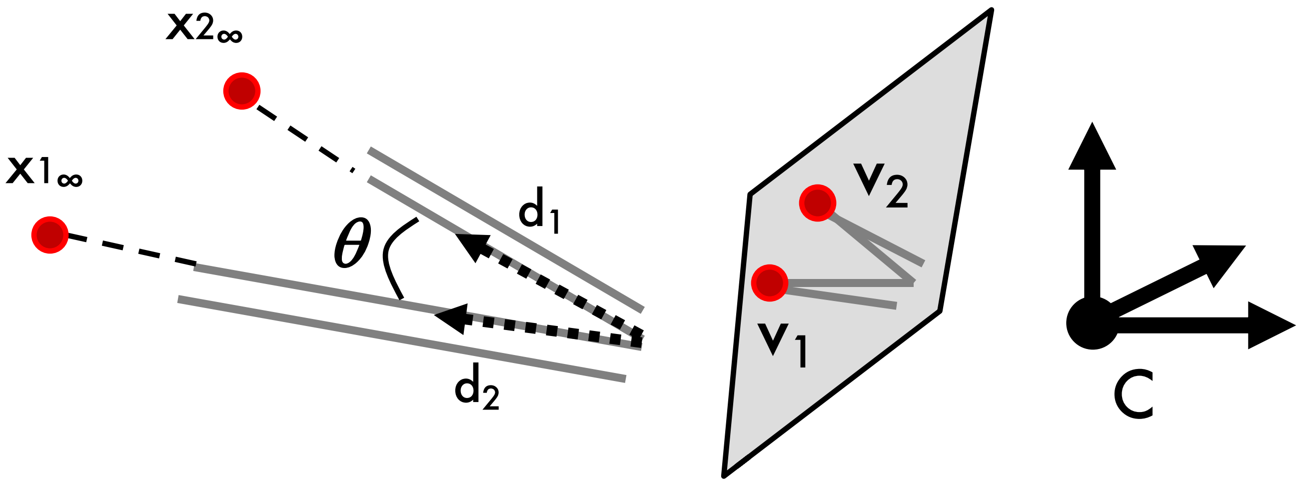


$$\Pi_\infty = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

plane at infinity

- Parallel planes intersect at infinity in a common line – **the line at infinity**
- A set of 2 or more lines at infinity defines the plane at infinity Π_∞

Angle between 2 vanishing points



$$\cos\theta = \frac{v_1^T \omega \, v_2}{\sqrt{v_1^T \omega \, v_1}\sqrt{v_2^T \omega \, v_2}}$$

[Eq. 28]

$$\omega = (K \, K^T)^{-1}$$

[Eq. 30]

If $\theta = 90 \rightarrow$ $v_1^T \omega \, v_2 = 0$ [Eq. 29]

Scalar equation

Properties of ω

$$\omega = (K\,K^T)^{-1}$$

[Eq. 30]

$$M = K\begin{bmatrix} R & T \end{bmatrix}$$

1. $$\omega = \begin{bmatrix} \omega_1 & \omega_2 & \omega_4 \\ \omega_2 & \omega_3 & \omega_5 \\ \omega_4 & \omega_5 & \omega_6 \end{bmatrix}$$ symmetric and known up scale

2. $\omega_2 = 0$ zero-skew

3. $\omega_2 = 0$, $\omega_1 = \omega_3$ square pixel

Summary

$$\mathbf{v} = K\,\mathbf{d}$$

[Eq. 24]

$$\mathbf{n} = \mathrm{K}^{\mathrm{T}}\mathbf{l}_{\mathrm{horiz}}$$

[Eq. 27]

$$\cos\theta = \frac{\mathrm{v}_1^{\mathrm{T}}\boldsymbol{\omega}\,\mathrm{v}_2}{\sqrt{\mathrm{v}_1^{\mathrm{T}}\boldsymbol{\omega}\,\mathrm{v}_1}\sqrt{\mathrm{v}_2^{\mathrm{T}}\boldsymbol{\omega}\,\mathrm{v}_2}}$$

[Eq. 28]

$$\xrightarrow{\theta = 90}$$

$$\mathrm{v}_1^{\mathrm{T}}\boldsymbol{\omega}\,\mathrm{v}_2 = 0$$

[Eq. 29]

$$\omega = (K\,K^T)^{-1}$$

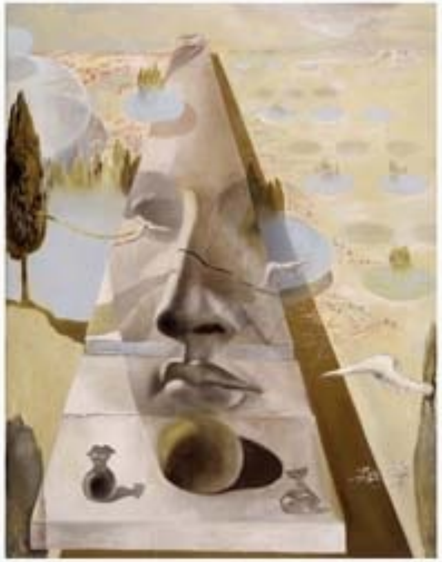
[Eq. 30]

Useful to:

- To calibrate the camera
- To estimate the geometry of the 3D world

Lecture 4

Single View Metrology



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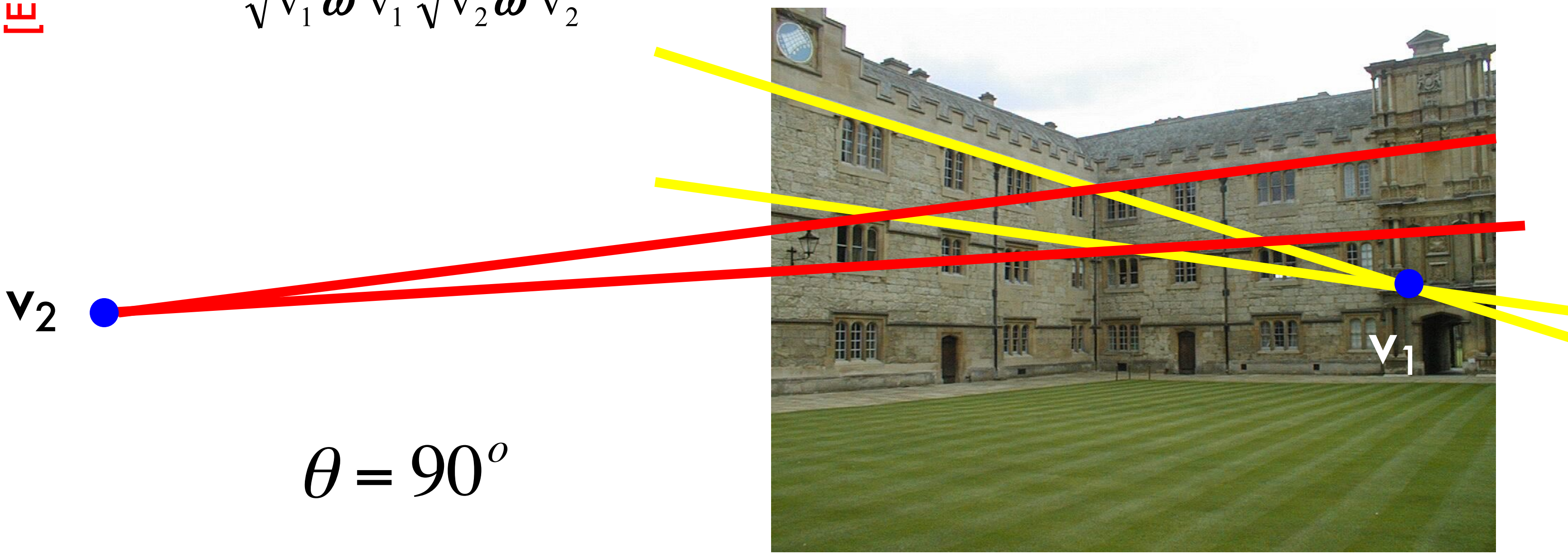
Single view calibration - example

[Eq. 28]

$$\cos\theta = \frac{v_1^T \omega \, v_2}{\sqrt{v_1^T \omega \, v_1}\sqrt{v_2^T \omega \, v_2}}$$



$$\theta = 90^o$$

[Eq. 29]

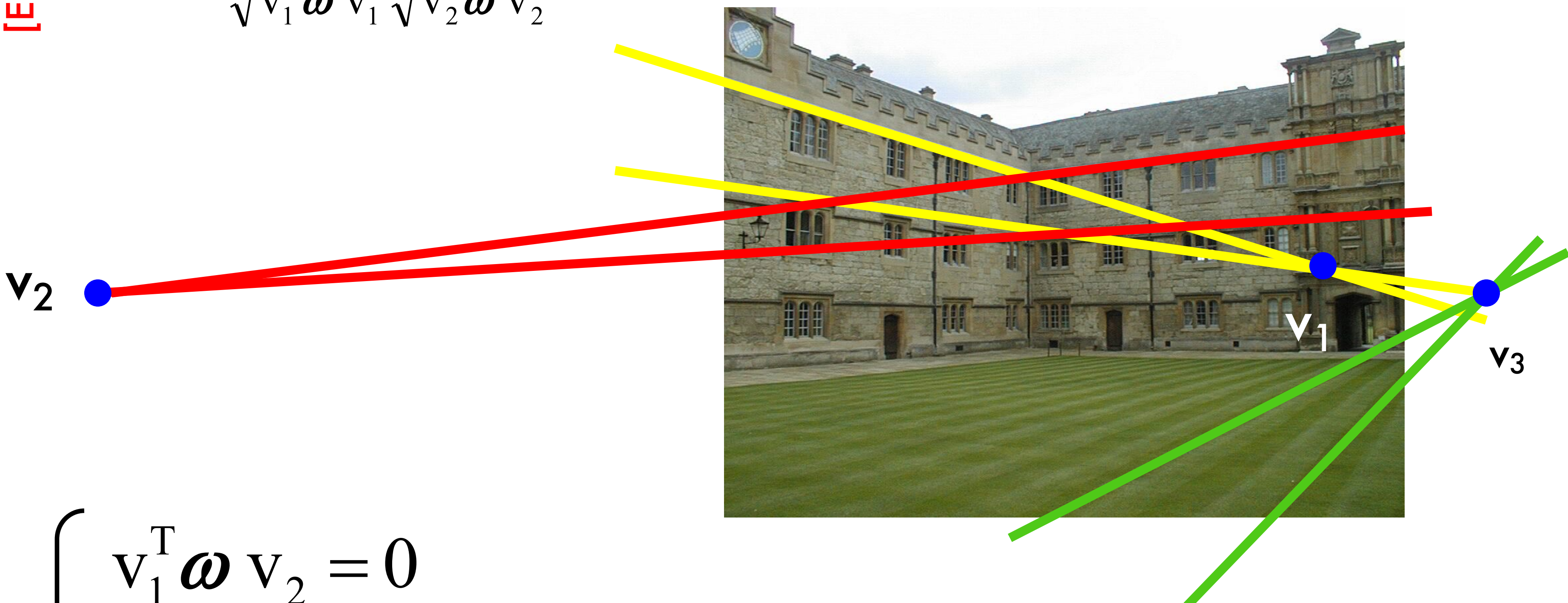
$$\begin{cases} v_1^T \omega \, v_2 = 0 \\ \omega = (K \, K^T)^{-1} \end{cases}$$

→ Do we have enough constraints to estimate K?
K has 5 degrees of freedom and Eq.29 is a scalar equation ☹

Single view calibration - example

[Eq. 28]

$$\cos\theta = \frac{v_1^T \omega\, v_2}{\sqrt{v_1^T \omega\, v_1}\sqrt{v_2^T \omega\, v_2}}$$



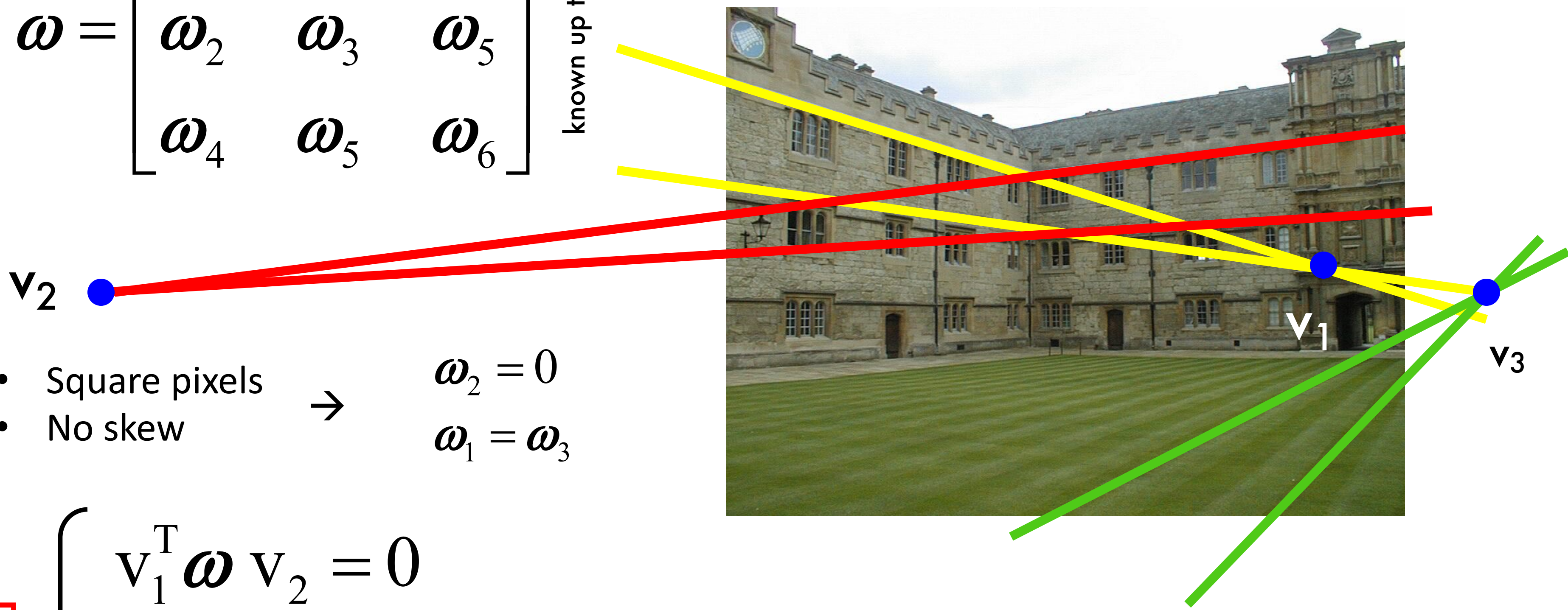
[Eqs. 31]

$$\begin{cases} v_1^T \omega\, v_2 = 0 \\ v_1^T \omega\, v_3 = 0 \\ v_2^T \omega\, v_3 = 0 \end{cases}$$

Single view calibration - example

$$\omega = \begin{bmatrix} \omega_1 & \omega_2 & \omega_4 \\ \omega_2 & \omega_3 & \omega_5 \\ \omega_4 & \omega_5 & \omega_6 \end{bmatrix}$$

known up to scale



- Square pixels
- No skew

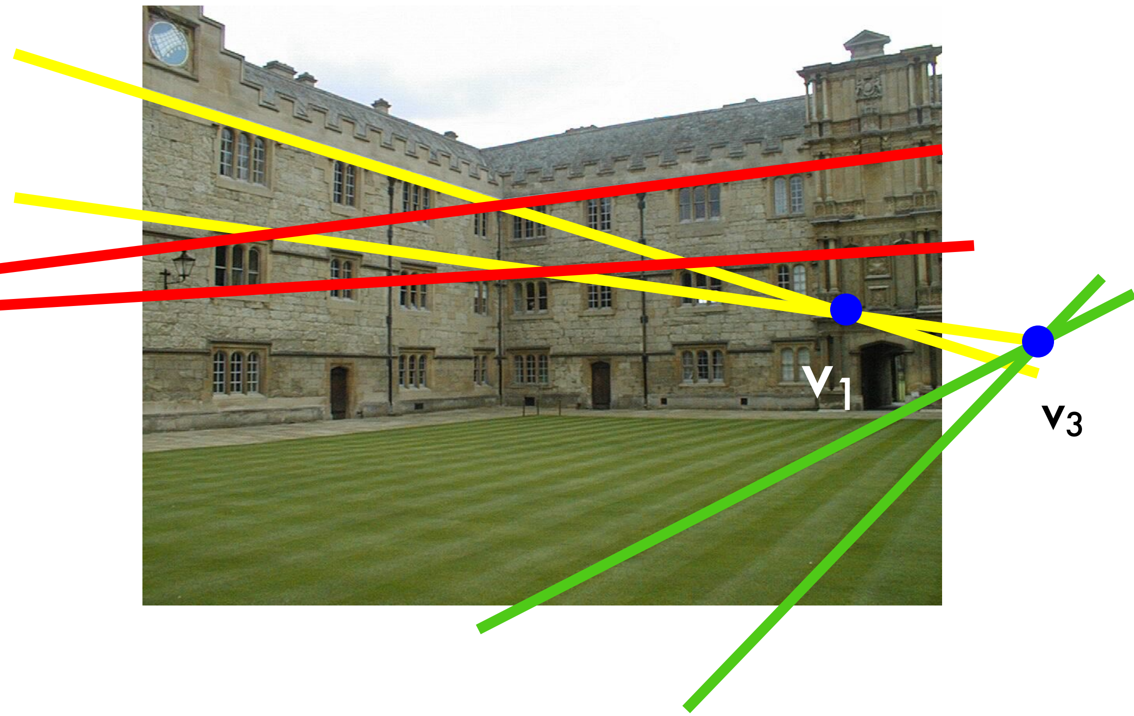
→ $\omega_2 = 0$, $\omega_1 = \omega_3$

[Eqs. 31]

$$\begin{cases} v_1^T \omega \, v_2 = 0 \\ v_1^T \omega \, v_3 = 0 \\ v_2^T \omega \, v_3 = 0 \end{cases}$$

Single view calibration - example

$$\omega = \begin{bmatrix} \omega_1 & 0 & \omega_4 \\ 0 & \omega_1 & \omega_5 \\ \omega_4 & \omega_5 & \omega_6 \end{bmatrix}$$ known up to scale



- Square pixels
- No skew

→ $\omega_2 = 0$, $\omega_1 = \omega_3$

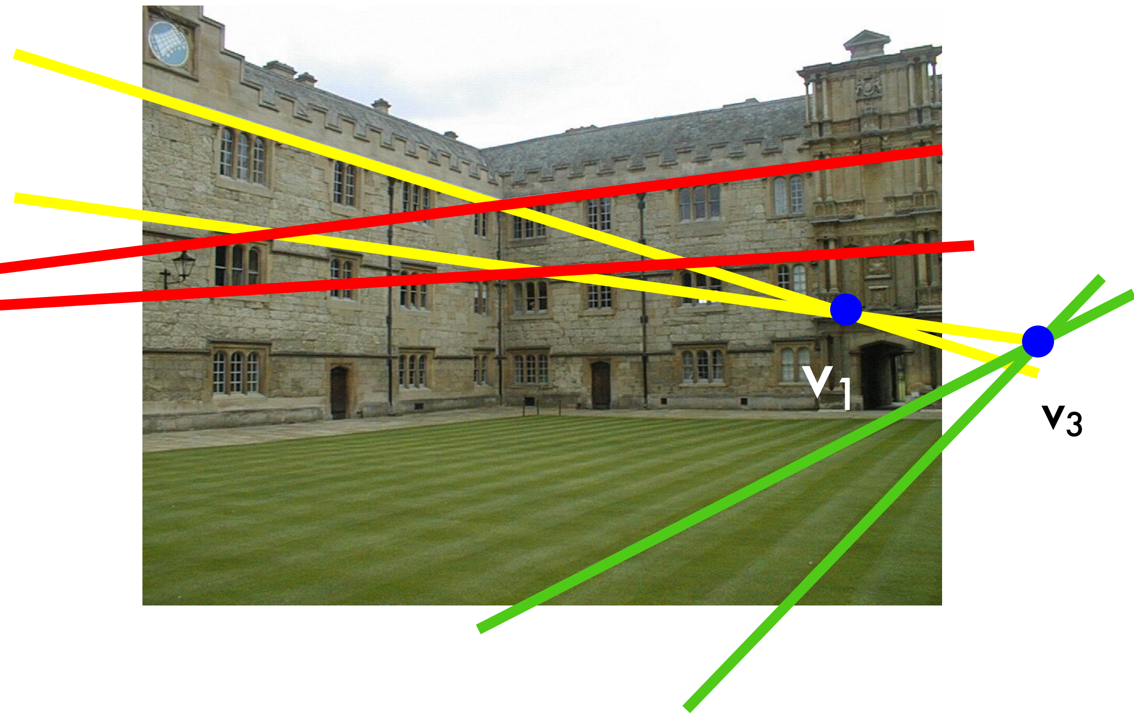
[Eqs. 31]

$$\begin{cases} v_1^T \omega \, v_2 = 0 \\ v_1^T \omega \, v_3 = 0 \\ v_2^T \omega \, v_3 = 0 \end{cases}$$

→ Compute ω !

Single view calibration - example

$$\omega = \begin{bmatrix} \omega_1 & 0 & \omega_4 \\ 0 & \omega_1 & \omega_5 \\ \omega_4 & \omega_5 & \omega_6 \end{bmatrix}$$



- Square pixels
- No skew

→ $\omega_2 = 0$, $\omega_1 = \omega_3$

[Eqs. 31]

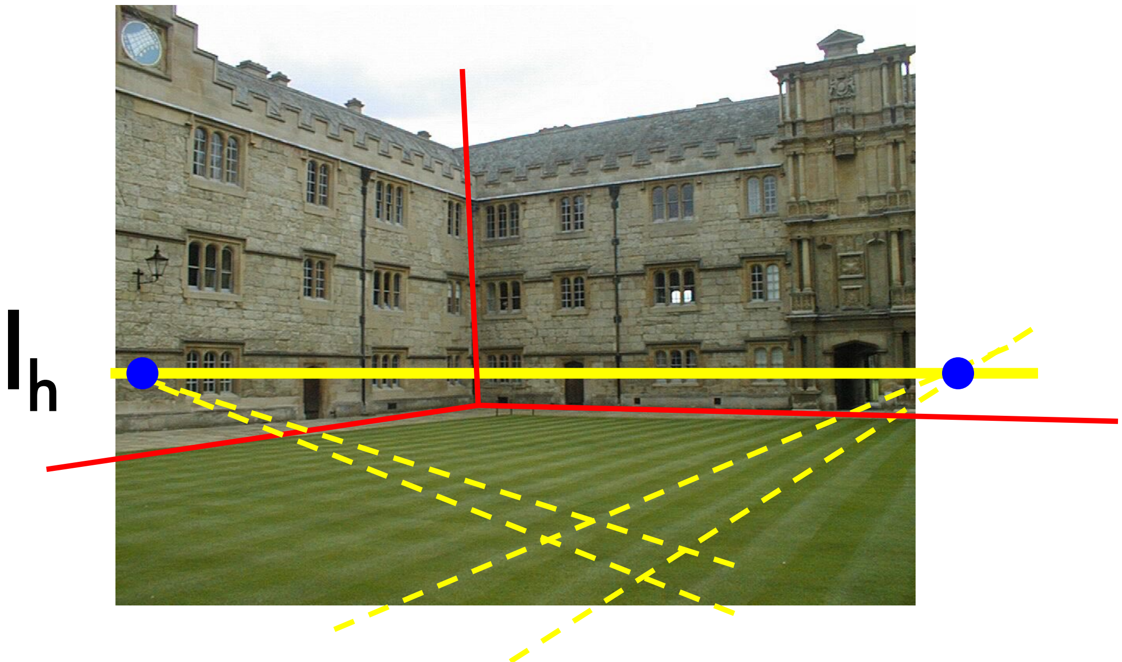
$$\begin{cases} v_1^T \omega \, v_2 = 0 \\ v_1^T \omega \, v_3 = 0 \\ v_2^T \omega \, v_3 = 0 \end{cases}$$

Once ω is calculated, we get K:

$$\omega = (K \, K^T)^{-1} \rightarrow K$$

(Cholesky factorization; HZ pag 582)

Single view reconstruction - example

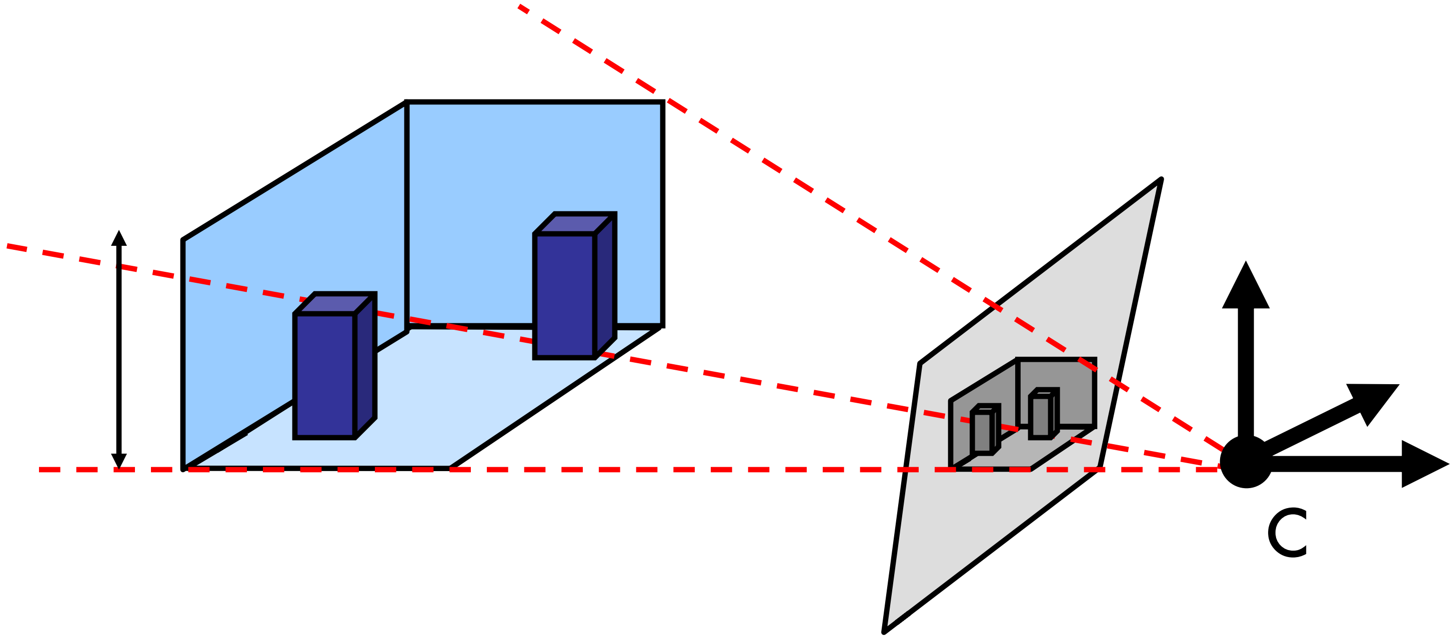


[Eq. 27]

K known $\rightarrow$ $$\mathbf{n} = K^T \mathbf{l}_{horiz}$$ = Scene plane orientation in the camera reference system

Select orientation discontinuities

Single view reconstruction - example



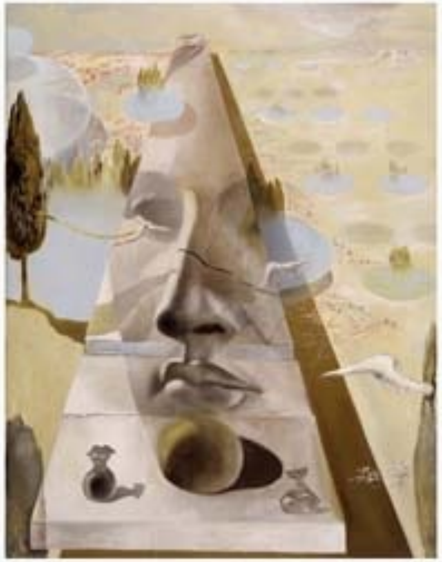
Recover the structure within the camera reference system

Notice: the actual scale of the scene is NOT recovered

- Recognition helps reconstruction!
- Humans have learnt this

Lecture 4

Single View Metrology



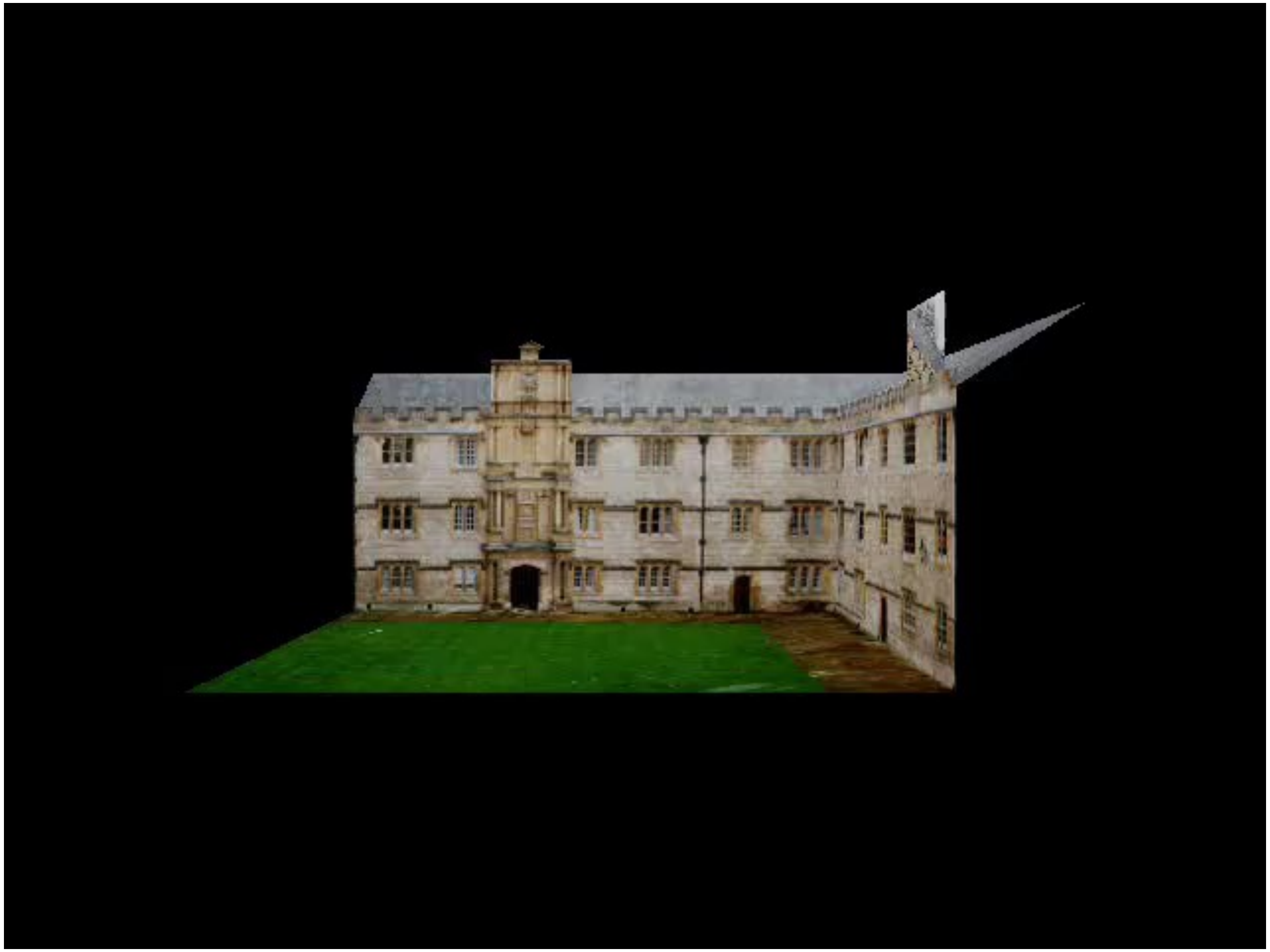
- Review calibration
- Vanishing points and lines
- Estimating geometry from a single image
- Extensions

Reading:
[HZ] Chapter 2 “Projective Geometry and Transformation in 3D”
[HZ] Chapter 3 “Projective Geometry and Transformation in 3D”
[HZ] Chapter 8 “More Single View Geometry”
[Hoeim & Savarese] Chapter 2

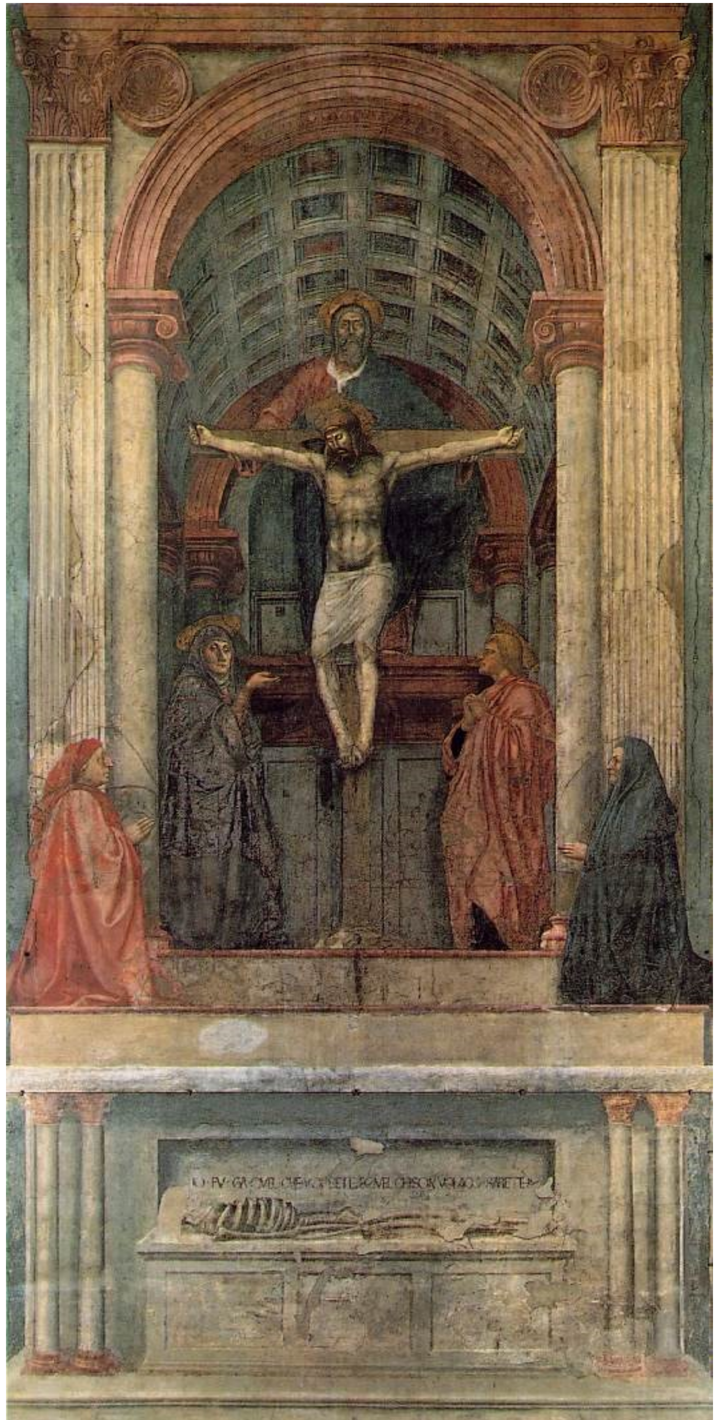
Criminisi & Zisserman, 99

http://www.robots.ox.ac.uk/~vgg/projects/SingleView/models/merton/merton.wrl

Criminisi & Zisserman, 99



http://www.robots.ox.ac.uk/~vgg/projects/SingleView/models/merton/merton.wrl

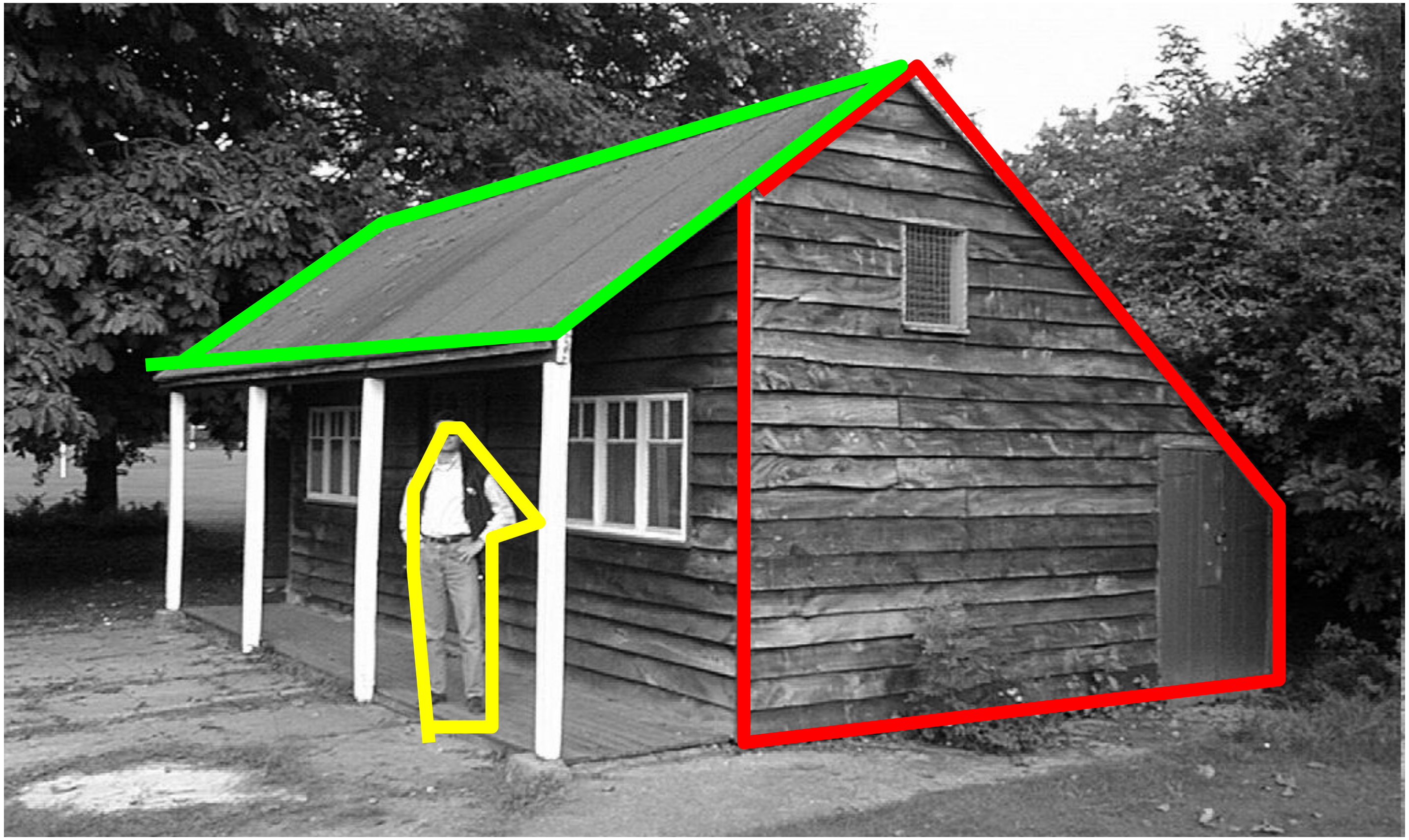


La Trinita' (1426)
Firenze, Santa Maria Novella; by Masaccio (1401-1428)

La Trinita' (1426)
Firenze, Santa Maria Novella; by Masaccio (1401-1428)

http://www.robots.ox.ac.uk/~vgg/projects/SingleView/models/hut/hutme.wrl

Single view reconstruction - drawbacks



Manually select:

- Vanishing points and lines;
- Planar surfaces;
- Occluding boundaries;
- Etc..

Automatic Photo Pop-up

Hoiem et al, 05

Automatic Photo Pop-up

Hoiem et al, 05…

Automatic Photo Pop-up

Hoiem et al, 05...

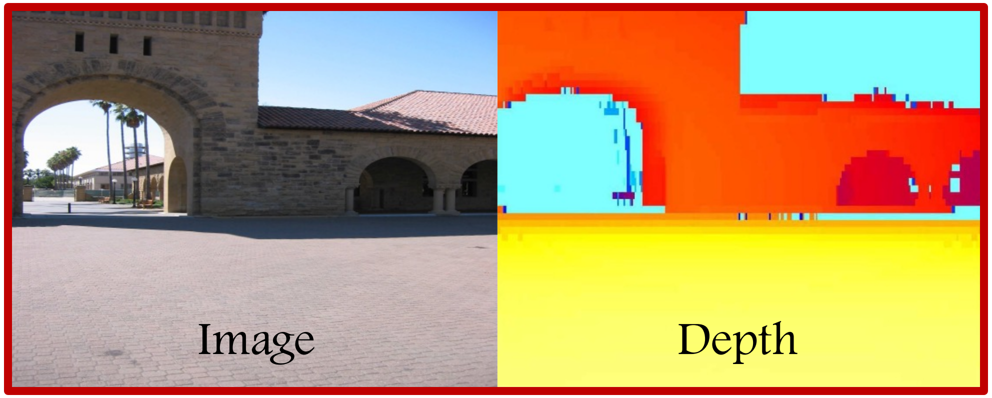
Software:

http://www.cs.uiuc.edu/homes/dhoiem/projects/software.html

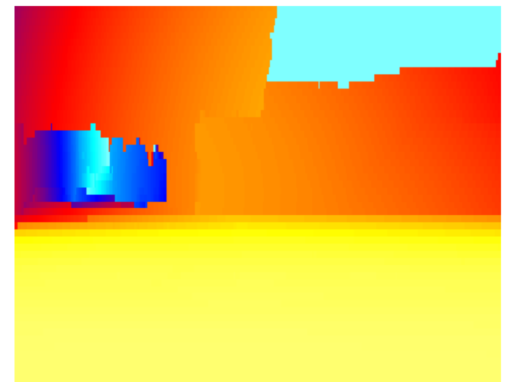
Make3D

Saxena, Sun, Ng, 05...

Training

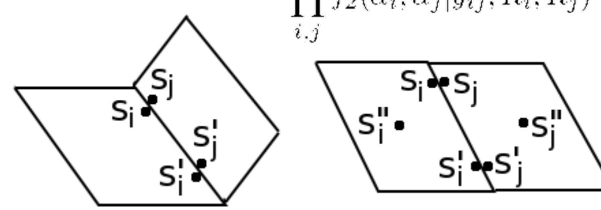


Prediction



Plane Parameter MRF

$$P(\alpha|X,\nu,y,R;\theta) = \frac{1}{Z}\prod_i f_1(\alpha_i|X_i,\nu_i,R_i;\theta) \prod_{i,j} f_2(\alpha_i,\alpha_j|y_{ij},R_i,R_j)$$



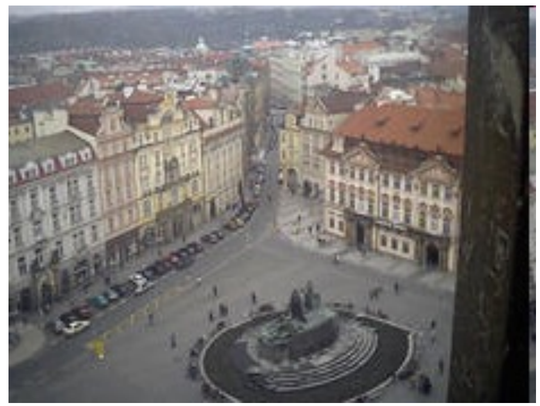
(a) Connectivity

(b) Co-Planarity

youtube

Make3D

Saxena, Sun, Ng, 05…


A software: **Make3D**
"Convert your image into 3d model"

http://make3d.stanford.edu/

http://make3d.cs.cornell.edu/

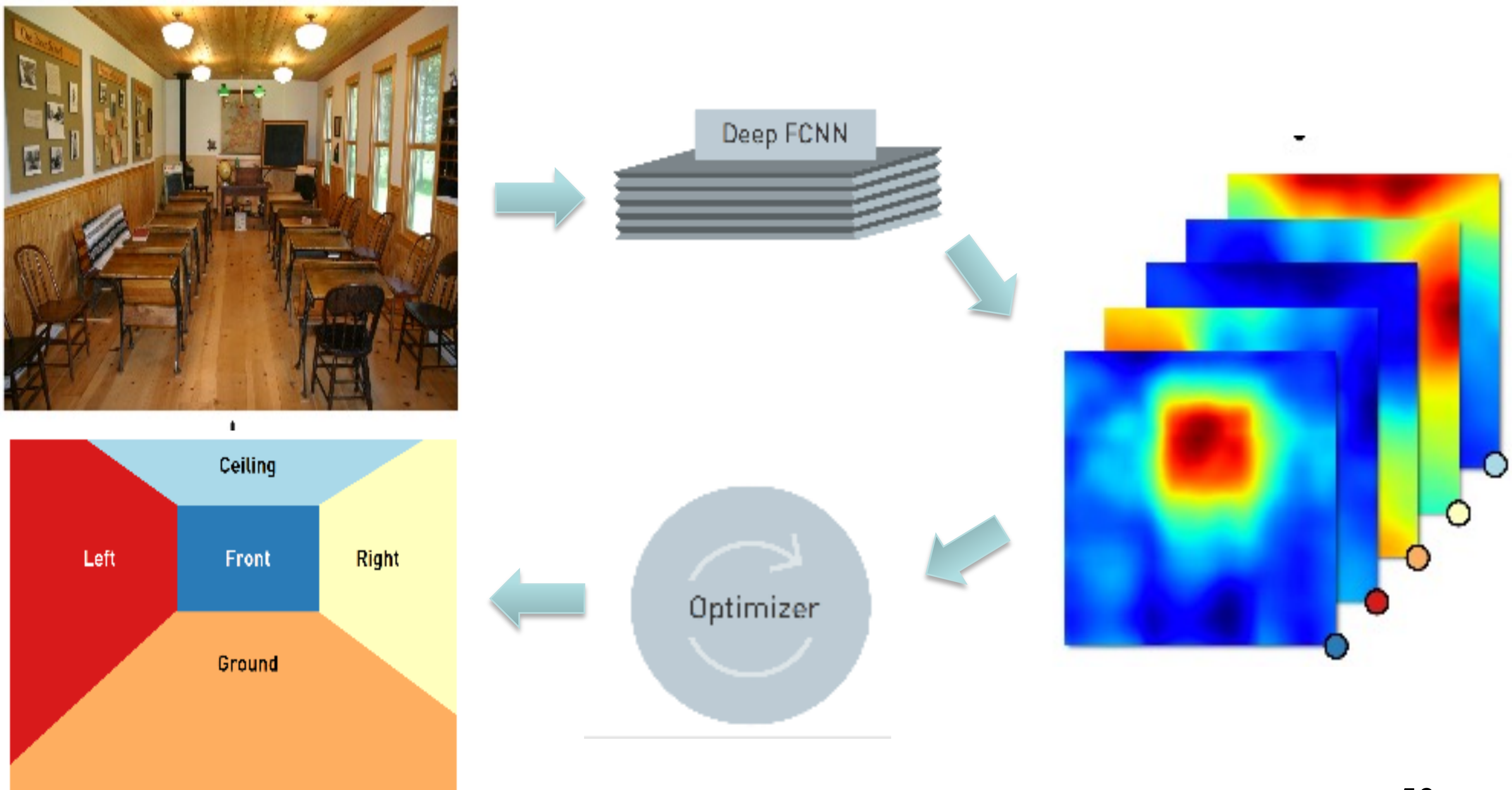
Depth map reconstruction using deep learning

Eigen et al., 2014

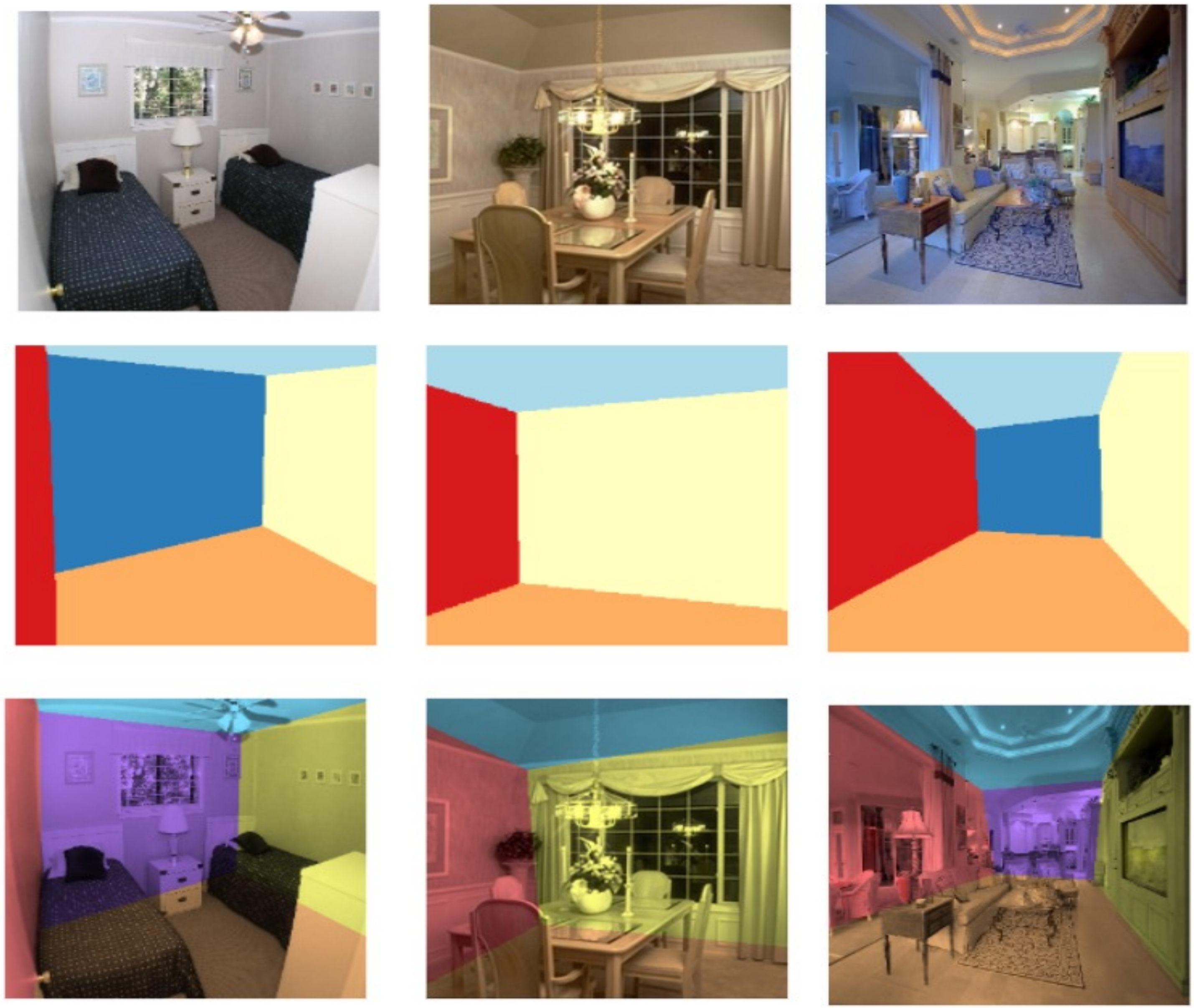
Depth Map Prediction from a Single Image using a Multi-Scale Deep Network,
Eigen, D., Puhrsch, C. and Fergus, R. Proc. Neural Information Processing Systems 2014,

3D Layout estimation

Dasgupta, et al. CVPR 2016

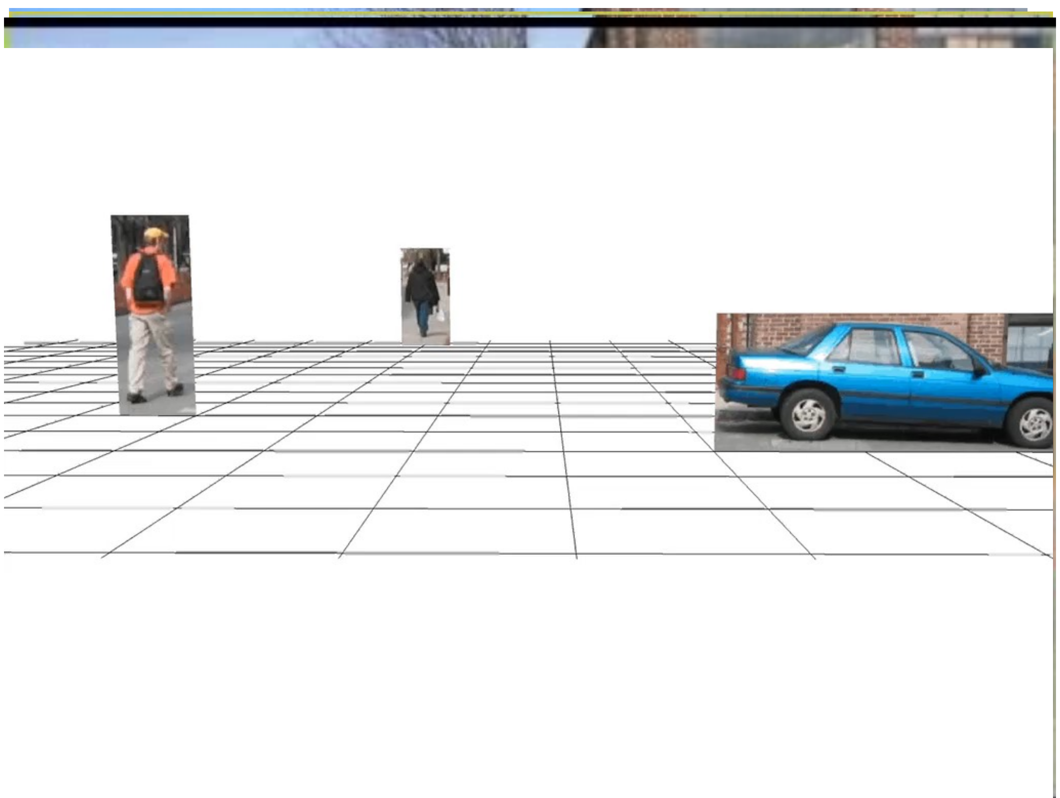


3D Layout estimation



Coherent object detection and scene layout estimation from a single image

Bao, et al., CVPR 2010, BMVC 2010



Next lecture:

Multi-view geometry (epipolar geometry)

Appendix